Chapter 7

Work and Kinetic Energy
Units of Chapter 7

• Work Done by a Constant Force
• Kinetic Energy and the Work-Energy Theorem
• Work Done by a Variable Force
• Power
7-1 Work Done by a Constant Force

The definition of work, when the force is parallel to the displacement:

\[ W = Fd \]  

(7-1)

SI unit: newton-meter (N·m) = joule, J
## 7-1 Work Done by a Constant Force

<table>
<thead>
<tr>
<th>Activity</th>
<th>Equivalent work (J)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Annual U. S. energy use</td>
<td>$8 \times 10^{19}$</td>
</tr>
<tr>
<td>Mt. St. Helens eruption</td>
<td>$10^{18}$</td>
</tr>
<tr>
<td>Burning one gallon of gas</td>
<td>$10^{8}$</td>
</tr>
<tr>
<td>Human food intake/day</td>
<td>$10^{7}$</td>
</tr>
<tr>
<td>Melting an ice cube</td>
<td>$10^{4}$</td>
</tr>
<tr>
<td>Lighting a 100-W bulb for 1 minute</td>
<td>6000</td>
</tr>
<tr>
<td>Heartbeat</td>
<td>0.5</td>
</tr>
<tr>
<td>Turning page of a book</td>
<td>$10^{-3}$</td>
</tr>
<tr>
<td>Hop of a flea</td>
<td>$10^{-7}$</td>
</tr>
<tr>
<td>Breaking a bond in DNA</td>
<td>$10^{-20}$</td>
</tr>
</tbody>
</table>
7-1 Work Done by a Constant Force

If the force is at an angle to the displacement:

$$W = (F \cos \theta)d = Fd \cos \theta$$  (7-3)
7-1 Work Done by a Constant Force

The work can also be written as the dot product of the force and the displacement:

\[ W = \mathbf{F} \cdot \mathbf{d} = Fd \cos \theta \]
7-1 Work Done by a Constant Force

The work done may be positive, zero, or negative, depending on the angle between the force and the displacement:

- \[ \theta < 90^\circ \]
  - \[ W > 0 \]

- \[ \theta = 90^\circ \]
  - \[ W = 0 \]

- \[ \theta > 90^\circ \]
  - \[ W < 0 \]
Example 7-2: Gravity Escape System. $m=4970\text{kg}$, $d=5\text{m}$, $h=2.5\text{m}$. How much work does gravity do on the escape boat as it slides down the ramp?
Example 7-2: Gravity Escape System. \( m = 4970 \text{ kg}, \ d = 5 \text{ m}, \ h = 2.5 \text{ m} \). How much work does gravity do on the escape boat as it slides down the ramp?

\[
W = F \cos \theta = mg \cos \theta = mg d \frac{h}{d} = mgh
\]

\[
W = (4970 \text{ kg})(9.81 \text{ m/s}^2)(2.5 \text{ m}) = 122,000 \text{ J}
\]

The work done does not depend on the angle!
Example: Path Dependence of Work. What is the work done on the box (a) when it is pushed up the ramp, (b) when it is raised straight up?

(a) \( W = F_1h = mgh \)

(b) \( W = F_2d = mgsin\phi L = mg \frac{h}{L} L = mgh \)

The force needed to push the box up the ramp with a constant velocity is equal in magnitude but opposite in direction to the component of gravity along the same direction.
7-1 Work Done by a Constant Force

If there is more than one force acting on an object, we can find the work done by each force, and also the work done by the net force:

\[ W_{\text{total}} = (F_{\text{total}} \cos \theta) d = F_{\text{total}} d \cos \theta \]  

(7-5)
Example 7-3: A coasting car. A car is acted on by air resistance, gravity and the normal force. What is the work done on the car as it travels a distance $d$?

$$ W = \text{sum of the work from each force} = W_N + W_{\text{air}} + W_g $$

$$ W_N = N \cos(90^\circ) = 0 $$

$$ W_{\text{air}} = F_{\text{air}} d \cos(180^\circ) = -F_{\text{air}} d $$

$$ W_g = mg d \cos(90^\circ - \phi) = mg d \sin \phi $$

$$ W = 0 - F_{\text{air}} d + mg d \sin \phi $$
Example 7-3: A coasting car. A car is acted on by air resistance, gravity and the normal force. What is the work done on the car as it travels a distance \(d\)?

\[
W = \text{(component of } F_{\text{total}} \text{ along motion})d
\]

\[
W = (mgsin\phi - F_{\text{air}})d
\]

The normal force does not have a component along the direction of motion.
7-2 Kinetic Energy

An object thrown up with an initial velocity $v_i$ will slow down to a velocity $v_f$ after travelling a distance $d$:

$v_f^2 = v_i^2 + 2ad \Rightarrow 2ad = v_f^2 - v_i^2 \Rightarrow 2 \frac{F}{m} d = v_f^2 - v_i^2 \Rightarrow$

$W = \frac{1}{2} mv_f^2 - \frac{1}{2} mv_i^2$

$E_K = \frac{1}{2} mv^2 = $ Kinetic Energy of an object with a mass $m$ and velocity $v$ (unit of J)

The work done on an object is equal to the change of its kinetic energy.
7-2 Kinetic Energy and the Work-Energy Theorem

Work-Energy Theorem: The total work done on an object is equal to its change in kinetic energy.

\[ W_{total} = \Delta K = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2 \]
Example 7-5 Hit the books. $m = 4.10\text{kg}$, $\Delta y = 1.60\text{m}$, $F = 52.7\text{ N}$

(a) Work done by the applied force?  
(b) Work done by gravity?  
(c) Final speed of box?
**Example 7-5** Hit the books. \( m = 4.10\text{kg}, \Delta y=1.60\text{m}, F=52.7\text{ N} \) (a) Work done by the applied force? (b) Work done by gravity? (c) Final speed of box?

(a) \( W_F = F_N \Delta y \cos(0^\circ) = (52.7 \text{ N})(1.60 \text{ m}) = 84.3 \text{ J} \)

(b) \( mg = (4.10\text{kg})(9.81\text{m/s}^2) = 40.22 \text{ J} \)

\( W_g = mg \Delta y \cos(180^\circ) = (40.22 \text{ N})(1.60 \text{ m}) = -64.35 \text{ J} \)

(c) Apply work-energy theorem:

\[
W_{\text{total}} = W_F + W_g = 84.3 \text{ J} - 64.35 \text{ J} = 20 \text{ W}
\]

\[
\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = \frac{1}{2}mv_f^2 - 0 \Rightarrow v_f = \left[ \frac{2}{4.10\text{kg}} (20 \text{ W}) \right]^{1/2} = 3.12 \text{ m/s}
\]
Example 7-6 Pulling a sled. $F = 11 \text{ N}$, $\theta = 29^\circ$, $m=6.40 \text{ kg}$, $v_i = 0.5 \text{ m/s}$ (a) Work done by boy ? (b) $v_f$ after 2 m

\[ W = Fd \cos(29^\circ) = (11.0 \text{ N})(2.00 \text{ m}) \cos(29^\circ) = 19.24 \text{ J} \]

Use the work - energy theorem

\[ W = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 \implies v_f = \sqrt{\frac{2W}{m}} + v_i^2 = 2.5 \text{ m/s} \]
7-3 Work Done by a Variable Force

If the force is constant, we can interpret the work done graphically:

\[ \text{Area} = Fd = W \]
7-3 Work Done by a Variable Force

If the force takes on several successive constant values:
7-3 Work Done by a Variable Force

We can then approximate a continuously varying force by a succession of constant values.
7-3 Work Done by a Variable Force

The force needed to stretch a spring an amount $x$ is $F = kx$.

Therefore, the work done in stretching the spring is

$$W = \frac{1}{2} kx^2 \quad (7-8)$$
7-3 Work Done by a Variable Force

Example 7-7 Flexing an AFM Cantilever. The work to deflect a cantilever by 0.1 nm is $W=1.2 \times 10^{-20}$ J

(a) What is the force constant of the cantilever?

(b) Work to deflect cantilever from 0.1 nm to 0.2 nm?

\[ W = \frac{1}{2} kd^2 \Rightarrow k = \frac{2W}{d^2} = 2.4 \text{ N/m} \]

\[ W_{0\rightarrow 2} = \frac{1}{2} (2.4 \text{ N/m})(2 \text{ nm})^2 = 4.8 \times 10^{-20} \text{ J} \]

\[ W_{1\rightarrow 2} = W_{0\rightarrow 2} - W_{0\rightarrow 1} = 4.8 \times 10^{-20} \text{ J} - 1.2 \times 10^{-20} \text{ J} = 3.6 \times 10^{-20} \text{ J} \]
7-3 Work Done by a Variable Force

Example A block compresses a spring. $m_{\text{block}} = 1.5 \text{ kg}$, $v_0=2.2 \text{ m/s}$. What is the compression of the spring when the spring comes to rest. $k = 475 \text{ N/m}$. 

---

Initial speed of block is $v_0$.

Equilibrium position of spring.

(a)

(c)

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7-3 Work Done by a Variable Force

Example A block compresses a spring. \( m_{\text{block}} = 1.5 \text{ kg} \), \( v_0 = 2.2 \text{ m/s} \). What is the compression of the spring when the spring comes to rest. \( k = 475 \text{ N/m} \).

\[
W = \Delta K = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2 = -\frac{1}{2} m v_i^2
\]

The work done by the spring on the block is negative since the force that slows the block down is in a direction opposite to the motion:

\[
W = -\frac{1}{2} k x^2 = -\frac{1}{2} m v_i^2 \quad \Rightarrow \quad x = \sqrt{\frac{m}{k} v_i} = \sqrt{\frac{(1.5 \text{ kg})}{475 \text{ N/m}} (2.2 \text{ m/s})} = 0.12 \text{ m}
\]
7-4 Power

Power is a measure of the rate at which work is done:

\[ P = \frac{W}{t} \]  \hspace{1cm} (7-10)

SI unit: J/s = watt, W

1 horsepower = 1 hp = 746 W
## 7-4 Power

### TABLE 7–3

<table>
<thead>
<tr>
<th>Source</th>
<th>Approximate power (W)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hoover Dam</td>
<td>$1.34 \times 10^9$</td>
</tr>
<tr>
<td>Car moving at 40 mph</td>
<td>$7 \times 10^4$</td>
</tr>
<tr>
<td>Home stove</td>
<td>$1.2 \times 10^4$</td>
</tr>
<tr>
<td>Sunlight falling on one square meter</td>
<td>1380</td>
</tr>
<tr>
<td>Refrigerator</td>
<td>615</td>
</tr>
<tr>
<td>Television</td>
<td>200</td>
</tr>
<tr>
<td>Person walking up stairs</td>
<td>150</td>
</tr>
<tr>
<td>Human brain</td>
<td>20</td>
</tr>
</tbody>
</table>
If an object is moving at a constant speed in the face of friction, gravity, air resistance, and so forth, the power exerted by the driving force can be written:

\[ P = \frac{Fd}{t} = F\left(\frac{d}{t}\right) = F\nu \]  
(7-13)
Example 7-8 Why you need a powerful car? To pass Joe you need to get your $m_{\text{car}} = 1.3 \times 10^3$ kg Porshe from $v_i = 13.4$ m/s to $v_f=17.9$ m/s in 3.00 sec. What is the minimum power required to pass?

\[ W = \Delta K = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = 9.16 \times 10^4 \text{ J} \]

\[ P = \frac{W}{t} = \frac{9.16 \times 10^4 \text{ J}}{3.00 \text{ sec}} \]
Summary of Chapter 7

• If the force is constant and parallel to the displacement, work is force times distance

• If the force is not parallel to the displacement,

\[ W = (F \cos \theta) d = Fd \cos \theta \]

• The total work is the work done by the net force:

\[ W_{\text{total}} = (F_{\text{total}} \cos \theta) d = F_{\text{total}} d \cos \theta \]
Summary of Chapter 7

• SI unit of work: the joule, J

• Total work is equal to the change in kinetic energy:

\[ W_{\text{total}} = \Delta K = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2 \]

where \( K = \frac{1}{2} m v^2 \)
Summary of Chapter 7

• Work done by a spring force:

\[ W = \frac{1}{2} kx^2 \]

• Power is the rate at which work is done:

\[ P = \frac{W}{t} \]

• SI unit of power: the watt, W