Chapter 15

Fluids
Units of Chapter 15

• Density
• Pressure
• Static Equilibrium in Fluids: Pressure and Depth
• Archimedes’ Principle and Buoyancy
• Applications of Archimedes’ Principle
• Fluid Flow and Continuity
Units of Chapter 15

- Bernoulli’s Equation
- Applications of Bernoulli’s Equation
- Viscosity and Surface Tension
15-1 Density

The density of a material is its mass per unit volume:

**Definition of Density, \( \rho \)**

\[ \rho = \frac{M}{V} \]

SI unit: kg/m\(^3\)

**TABLE 15–1**

Densities of Common Substances

<table>
<thead>
<tr>
<th>Substance</th>
<th>Density (kg/m(^3))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gold</td>
<td>19,300</td>
</tr>
<tr>
<td>Mercury</td>
<td>13,600</td>
</tr>
<tr>
<td>Lead</td>
<td>11,300</td>
</tr>
<tr>
<td>Silver</td>
<td>10,500</td>
</tr>
<tr>
<td>Iron</td>
<td>7860</td>
</tr>
<tr>
<td>Aluminum</td>
<td>2700</td>
</tr>
<tr>
<td>Ebony (wood)</td>
<td>1220</td>
</tr>
<tr>
<td>Ethylene glycol (antifreeze)</td>
<td>1114</td>
</tr>
<tr>
<td>Whole blood (37 °C)</td>
<td>1060</td>
</tr>
<tr>
<td>Seawater</td>
<td>1025</td>
</tr>
<tr>
<td>Freshwater</td>
<td>1000</td>
</tr>
<tr>
<td>Olive oil</td>
<td>920</td>
</tr>
<tr>
<td>Ice</td>
<td>917</td>
</tr>
<tr>
<td>Ethyl alcohol</td>
<td>806</td>
</tr>
<tr>
<td>Cherry (wood)</td>
<td>800</td>
</tr>
<tr>
<td>Balsa (wood)</td>
<td>120</td>
</tr>
<tr>
<td>Styrofoam</td>
<td>100</td>
</tr>
<tr>
<td>Oxygen</td>
<td>1.43</td>
</tr>
<tr>
<td>Air</td>
<td>1.29</td>
</tr>
<tr>
<td>Helium</td>
<td>0.179</td>
</tr>
</tbody>
</table>
15-1 Density

Example: Compare the weight of a dozen eggs to the weight of air in a refrigerator. Assume each egg has a mass of 44 g, $\rho_{\text{air}} = 1.29\text{kg/m}^3$ and the refrigerator has a volume of 0.45 m$^3$.

$$m_{\text{eggs}} = 12(44\text{g}) = 0.53\text{ kg}$$

$$m_{\text{air}} = \rho V = (1.29\text{kg/m}^3)(0.45\text{m}^3) = 0.58\text{kg}$$

The air weighs slightly more than the eggs!
15-2 Pressure

Pressure is force per unit area:

**Definition of Pressure, \( P \)**

\[ P = \frac{F}{A} \]

SI unit: \( \text{N/m}^2 \)
15-2 Pressure

The same force applied over a smaller area results in greater pressure – think of poking a balloon with your finger and then with a needle.
15-2 Pressure

Atmospheric pressure is due to the weight of the atmosphere above us.

\[
\text{Atmospheric Pressure, } P_{\text{at}} = 1.01 \times 10^5 \text{ N/m}^2
\]

SI unit: \text{N/m}^2

The pascal (Pa) is 1 \text{ N/m}^2. Pressure is often measured in pascals.
15-2 Pressure

There are a number of different ways to describe atmospheric pressure.

In pascals: \( P_{at} = 101 \text{ kPa} \)

In pounds per square inch: \( P_{at} = 14.7 \text{ lb/in}^2 \)

In bars:

\[ 1 \text{ bar} = 10^5 \text{ Pa} \approx 1 P_{at} \]
15-2 Pressure

Example 15-1: Air Pressure. Calculate the force on the palm (0.08 m × 0.1 m) of your hand from atmospheric pressure.

\[ F = PA = (1.01 \times 10^5 \text{N/m}^2)(0.08\text{m})(0.1\text{m}) = 810\text{N} \]

This force is applied on your hand from all direction so there is not net force.
15-2 Pressure

Since atmospheric pressure acts uniformly in all directions, we don’t usually notice it.

Therefore, if you want to, say, add air to your tires to the manufacturer’s specification, you are not interested in the total pressure. What you are interested in is the gauge pressure – how much more pressure is there in the tire than in the atmosphere?

\[ P_g = P - P_{at} \]
15-2 Pressure

Example 15-2 Pushing down on a basketball with a force of 22N you notice the area of contact becomes a circle with a radius of 1cm. Estimate the pressure in the basketball.
15-2 Pressure

Example 15-2 Pushing down on a basket ball with a force of 22N you notice the area of contact becomes a circle with a radius of 1cm. Estimate the pressure in the basket ball.

The pressure in the ball is $P = P_{\text{atm}} + P_{\text{gauge}}$

When we apply a force on the ball it is applied to the flat surface over an area of $A$. The pressure of this force on the surface is:

$$P_{\text{surface}} = \frac{F}{A}$$

This pressure is equal to the gauge pressure of the ball

$$P_{\text{gauge}} = P_{\text{surface}} = \frac{F}{A} = \frac{22\text{N}}{\pi(0.01\text{m})^2} = 7 \times 10^4 \text{Pa}$$

$$P = P_{\text{atm}} + P_{\text{gauge}} = 1.01 \times 10^5 \text{Pa} + 7 \times 10^4 \text{Pa}$$
15-3 Static Equilibrium in Fluids: Pressure and Depth

The increased pressure as an object descends through a fluid is due to the increasing mass of the fluid above it.

\[ F_{\text{top}} = P_{\text{atm}} A \]

\[ F_{\text{bottom}} = F_{\text{top}} + mg = F_{\text{top}} + \rho V g = F_{\text{top}} + \rho A h g \]

\[ P_{\text{bottom}} = \frac{F_{\text{bottom}}}{A} = \frac{F_{\text{top}} + \rho A h g}{A} = P_{\text{atm}} + \rho h g \]
15-3 Static Equilibrium in Fluids: Pressure and Depth

Example: Pressure on the Titanic. The Titanic was found in 1985 at a depth of 2.5 miles. What is the pressure at this depth?

\[
P_{\text{bottom}} = P_{\text{atm}} + \rho gh = \\
1.01 \times 10^5 \text{Pa} + (1025 \text{ kg/m}^3)(2.5 \times 1609.344 \text{m})(9.81 \text{m/s}^2) = 4.1 \times 10^7 \text{Pa}
\]
Example 15-3: The pressure at the top of a cubical box (20cm on a side) immersed in a fluid is 105 kPa and at the bottom 106.6 kPa. What is the density of the fluid.
15-3 Static Equilibrium in Fluids: Pressure and Depth

**Dependence of Pressure on Depth**

\[ P_2 = P_1 + \rho gh \]
Example 15-3 : The pressure at the top of a cubical box (20cm on a side) immersed in a fluid is 105 kPa and at the bottom 106.6kPa. What is the density of the fluid.

\[
\rho = \frac{P_{\text{bottom}} - P_{\text{top}}}{hg} \Rightarrow \rho = \frac{106.8 \text{ kPa} - 105 \text{ kPa}}{(0.2\text{m})(9.81\text{m/s}^2)} = 920 \text{ kg/m}^3
\]
Example 15-3: As the bubbles rise what happens to their diameter and why?
15-3 Static Equilibrium in Fluids: Pressure and Depth

A barometer compares the pressure due to the atmosphere to the pressure due to a column of fluid, typically mercury. The mercury column has a vacuum above it, so the only pressure is due to the mercury itself.
15-3 Static Equilibrium in Fluids: Pressure and Depth

This leads to the definition of atmospheric pressure in terms of millimeters of mercury:

In the barometer, the level of mercury is such that the pressure due to the column of mercury is equal to the atmospheric pressure.
15-3 Static Equilibrium in Fluids: Pressure and Depth

At any height in the tube shown the pressure is: \( P_{\text{atm}} + \) the pressure from the liquid above that height. Find \( h \).
15-3 Static Equilibrium in Fluids: Pressure and Depth

At any height in the tube shown the pressure is: \( P_{\text{atm}} + \) the pressure from the liquid above that height.

\[
\begin{align*}
P_C &= P_{\text{atm}} + \rho_w gh_1 + \rho g h_{AC} \\
P_D &= P_{\text{atm}} + \rho_{\text{oil}} gh_2 + \rho g h_{BD}
\end{align*}
\]

\[
P_C = P_D \implies P_{\text{atm}} + \rho_w gh_1 + \rho g h_{AC} = P_{\text{atm}} + \rho_{\text{oil}} gh_2 + \rho g h_{BD} \implies
\]

\[
\rho_w gh_1 = \rho_{\text{oil}} gh_2 \implies \rho_w (h_2 - h) = \rho_{\text{oil}} h_2 \implies
\]

\[
\rho_w h = \rho_w h_2 - \rho_{\text{oil}} h_2 \implies h = h_2 \left(1 - \frac{\rho_{\text{oil}}}{\rho_w}\right) = 0.4 \text{ cm}
\]
Pascal’s principle:

An external pressure applied to an enclosed fluid is transmitted unchanged to every point within the fluid. Find $F_2$. 
15-3 Static Equilibrium in Fluids: Pressure and Depth

We apply Pascal's Principle:
The pressure applied to the fluid from the piston on the left is transmitted unchanged to the entire fluid:

$$\Delta P = \frac{F_1}{A_1}$$

The force on the piston on the right is:

$$F_2 = (\Delta P)A_2 = F_1 \frac{A_2}{A_1}$$

Since the fluid does not compress we can determine the distance $d_2$ piston 2 moves when piston 1 moves by a distance of $d_1$.

$$V = d_1 A_1 = d_2 A_2 \Rightarrow d_2 = d_1 \frac{A_1}{A_2}$$
A fluid exerts a net upward force on any object it surrounds, called the **buoyant force**.

This force is due to the increased pressure at the bottom of the object compared to the top.

\[ F_b = F_2 - F_1 = \rho g L^3 \]
Archimedes’ Principle: An object completely immersed in a fluid experiences an upward buoyant force equal in magnitude to the weight of fluid displaced by the object.

Buoyant Force When a Volume $V$ Is Submerged in a Fluid of Density $\rho$

$$F_b = \rho g V$$

SI unit: N
Example 15-5 A person who weighs 720N is submerged in water. His apparent weight (scale reading) submerged is 34.3N. Find (a) his volume, (b) his density.
15-4 Archimedes’ Principle and Buoyancy

Example 15-5 A person who weighs 720N is submerged in water. His apparent weight (scale reading) submerged is 34.3N. Find (a) his volume, (b) his density.

\[ \sum F = 0 \Rightarrow F_b + W_a = mg, \text{ where } F_b \text{ is the buoyant force and } W_a \text{ is his apparent weight (scale reading)} \]

\[ F_b = \text{weight of displaced water} = V \rho_w g \]

\[ (a) \ V \rho_w g + W_a = mg \Rightarrow V = \frac{mg - W_a}{\rho_w g} = \frac{720 \text{ N} - 34.3 \text{ N}}{(1000 \text{ kg/m}^3)9.81 \text{m/s}^2} = 6.99 \times 10^{-2} \text{ m}^3 \]

\[ (b) \ \rho = \frac{m}{V} = \frac{W}{g V} = \frac{W}{V g} = \frac{720 \text{ N}}{(6.99 \times 10^{-2} \text{ m}^3)} = 1050 \text{ kg/m}^3 \]
15-4 Archimedes’ Principle and Buoyancy

Example: What is the percentage of a person's body-fat if they have a density of $\rho_p$. Assume that body fat has a density of $\rho_f = 900 \text{ kg/m}^3$ and lean body mass has a density of $\rho_l = 1100 \text{ kg/m}^3$.

The percentage of body fat is $x_f = \frac{m_f}{M}$, $M$ is the total mass and $m_f$ is the body-fat mass.

$$
\rho_f = \frac{m_f}{V_f} \Rightarrow m_f = V_f \rho_f = \left( V - V_l \right) \rho_f = \left( V - \frac{m_l}{\rho_l} \right) \rho_f = \left( V - \frac{M - m_f}{\rho_l} \right) \rho_f = V \rho_f - \frac{M}{\rho_l} \rho_f + \frac{m_f \rho_f}{\rho_l}
$$

$$
m_f \left( 1 - \frac{\rho_f}{\rho_l} \right) = \rho_f \left( V - \frac{M}{\rho_l} \right) \Rightarrow m_f = \frac{\rho_f \left( V - \frac{M}{\rho_l} \right)}{1 - \frac{\rho_f}{\rho_l}}
$$

$$
x_f = \frac{m_f}{M} = \frac{\rho_f \left( V - \frac{1}{\rho_l} \right)}{\left( 1 - \frac{\rho_f}{\rho_l} \right)} = \frac{\rho_f \left( \frac{1}{\rho_p} - \frac{1}{\rho_l} \right)}{\left( 1 - \frac{\rho_f}{\rho_l} \right)} = \frac{\rho_f - \rho_f}{\rho_p - \rho_l} \Rightarrow x_f = \frac{4950 \text{ kg/m}^3}{\rho_p} - 4.5 \text{ (Siri's Formula)}
$$
An object floats when the buoyant force is equal to its weight.

This is equivalent to:

An object floats when the weight of the displaced water is equal to the weight of the object.
Example: **Tension in string.** A piece of wood with a density of 706 kg/m³ and a volume of 8×10⁻⁶ m³ is submerged in water and tied to a string from the bottom. What is the tension of the string?
15-5 Applications of Archimedes’ Principle

Example: Tension in string. A piece of wood with a density of 706 kg/m³ and a volume of \(8 \times 10^{-6}\) m³ is submerged in water and tied to a string from the bottom. What is the tension of the string?

\[
\sum F = 0 \Rightarrow F_b - T - mg = 0 \Rightarrow T = F_b - mg
\]

\[
F_b = \rho_{\text{water}} V g
\]

\[
m = \rho_{\text{wood}} V
\]

\[
T = \rho_{\text{water}} V g - \rho_{\text{wood}} V g \Rightarrow
\]

\[
T = gV(\rho_{\text{water}} - \rho_{\text{wood}})
\]

Tension is positive since the density of water is larger than the density of wood.
Example: How much water is displaced by a cubical piece of wood that is 15 cm on a side. $\rho_{\text{wood}} = 655 \text{ kg/m}^3$.

When the piece of wood floats the buoyant force is equal to the weight of the wood. $\Rightarrow$

$$F_b = mg \Rightarrow \rho_{\text{water}} V_d g = mg = \rho_{\text{wood}} V_{\text{wood}} g \Rightarrow$$

$$V_d = \frac{\rho_{\text{wood}} V_{\text{wood}}}{\rho_{\text{water}}} = 2.21 \times 10^{-3} \text{ m}^3$$

where $V_d$ is the volume of displaced water.
An object made of material that is denser than water can float only if it has indentations or pockets of air that make its average density less than that of water.
15-5 Applications of Archimedes’ Principle

The fraction of an object that is submerged when it is floating depends on the densities of the object and of the fluid.

**Submerged Volume** $V_{\text{sub}}$ for a Solid of Volume $V_s$ and Density $\rho_s$ Floating in a Fluid of Density $\rho_f$

$$V_{\text{sub}} = V_s \left( \frac{\rho_s}{\rho_f} \right)$$

SI unit: $m^3$
Continuity tells us that whatever the mass of fluid in a pipe passing a particular point per second, the same mass must pass every other point in a second. The fluid is not accumulating or vanishing along the way.

This means that where the pipe is narrower, the flow is faster, as everyone who has played with the spray from a drinking fountain well knows.
Conservation of mass indicates that the mass of fluid passing at point (1) in time $\Delta t$ is equal to the mass of fluid passing at point (2) in time $\Delta t$.

**Equation of Continuity**

$$\rho_1 A_1 v_1 = \rho_2 A_2 v_2$$
Most gases are easily compressible; most liquids are not. Therefore, the density of a liquid may be treated as constant, but not that of a gas.

**Equation of Continuity for an Incompressible Fluid**

\[ A_1 v_1 = A_2 v_2 \]
Example 15-7 Spray 1 Water travels through a 9.6 cm diameter fire hose with a speed of 1.3 m/s. At the end of the hose the water flows out a nozzle whose diameter is 2.5 cm. What is the speed of the water coming out.
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The continuity equation for fluids is:

\[ \rho A v \text{ is constant along the flow.} \]

\[ \rho_1 A_1 v_1 = \rho_2 A_2 v_2, \text{ since the fluid is incompressible } \rho_1 = \rho_2 \]

\[ A_1 v_1 = A_2 v_2 \implies v_2 = \frac{A_1}{A_2} v_1 = \frac{\pi \left(\frac{d_1}{2}\right)^2}{\pi \left(\frac{d_2}{2}\right)^2} v_1 = \left(\frac{d_1}{d_2}\right)^2 v_1 \implies \]

\[ v_2 = \left(\frac{9 \text{ cm}}{2.5 \text{ cm}}\right)^2 (1.3 \text{ m/s}) = 19 \text{ m/s} \]
15-7 Bernoulli’s Equation

When a fluid moves from a wider area of a pipe to a narrower one, its speed increases; therefore, work has been done on it.

\[ \Delta W_{\text{total}} = (P_1 - P_2) \Delta V \]
15-7 Bernoulli’s Equation

The kinetic energy of a fluid element is:

\[ K = \frac{1}{2}(\Delta m)v^2 = \frac{1}{2}(\rho \Delta V)v^2 \]

Equating the work done to the increase in kinetic energy gives:

\[ P_1 + \frac{1}{2} \rho v_1^2 = P_2 + \frac{1}{2} \rho v_2^2 \]
15-7 Bernoulli’s Equation

If a fluid flows in a pipe of constant diameter, but changes its height, there is also work done on it against the force of gravity.

Equating the work done with the change in potential energy gives:

\[ P_1 + \rho g y_1 = P_2 + \rho g y_2 \]
15-7 Bernoulli’s Equation

The general case, where both height and speed may change, is described by Bernoulli’s equation:

\[
\text{Bernoulli’s Equation} \\
\frac{1}{2} \rho v_1^2 + \rho g y_1 = \frac{1}{2} \rho v_2^2 + \rho g y_2
\]

This equation is essentially a statement of conservation of energy in a fluid.
Example 15-7 Spray 2 Suppose the pressure in the fire hose is 350 kPa. Find the pressure in the nozzle.

The Bernoulli Equation for fluids is:

\[ P + \frac{1}{2} \rho v^2 + \rho gh \text{ is constant along the flow.} \]

\[ P_1 + \frac{1}{2} \rho v_1^2 + \rho gh = P_2 + \frac{1}{2} \rho v_2^2 + \rho gh \Rightarrow \]

\[ P_1 + \frac{1}{2} \rho v_1^2 = P_2 + \frac{1}{2} \rho v_2^2 \Rightarrow P_2 = P_1 + \frac{1}{2} \rho v_1^2 - \frac{1}{2} \rho v_2^2 \]

\[ P_2 = P_1 + \frac{1}{2} \rho \left( v_1^2 - v_2^2 \right) = P_1 + \frac{1}{2} \rho v_1^2 \left( 1 - \frac{v_2^2}{v_1^2} \right) = P_1 + \frac{1}{2} \rho v_1^2 \left( 1 - \left( \frac{d_1}{d_2} \right)^4 \right) \]

\[ P_2 = 170 \text{ kPa} \]
Example  **Find The Pressure** Water flows through a garden hose that goes up a step 20 cm high. If the water pressure is 143 kPa at the bottom of the step, what is the pressure at the top of the step.

The Bernoulli Equation for fluids is:

\[ P + \frac{1}{2} \rho v^2 + \rho gh \text{ is constant along the flow.} \]

\[ P_1 + \frac{1}{2} \rho v^2 + \rho g h_1 = P_2 + \frac{1}{2} \rho v^2 + \rho g h_2 \Rightarrow \]

\[ P_1 + \rho g h_1 = P_2 + \rho g h_2 \Rightarrow P_2 = P_1 + \rho g h_1 - \rho g h_2 \]

\[ P_2 = P_1 + \rho g (h_1 - h_2) = 143 \text{ kPa } + (1 \times 10^3 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(0 - 0.2 \text{ m}) \]

\[ P_2 = 141 \text{ kPa} \]
15-7 Bernoulli’s Equation

Example  **Find The Pressure** If the water pressure is 143 kPa at the bottom of the step, what is the pressure at the top of the step. The cross-sectional area of the hose on top is half that at the bottom and the speed at the bottom is 1.2 m/s.

From continuity: \( A_1 v_1 = A_2 v_2 \Rightarrow v_2 = \frac{A_1}{A_2} v_1 \)

\[
P_1 + \frac{1}{2} \rho v_1^2 + \rho gh_1 = P_2 + \frac{1}{2} \rho v_2^2 + \rho gh_2 \Rightarrow
\]

\[
P_2 = P_1 + \frac{1}{2} \rho (v_1^2 - v_2^2) + \rho g (h_1 - h_2)
\]

\[
P_2 = P_1 + \frac{1}{2} \rho \left(v_1^2 - \left(\frac{A_1}{A_2} v_1\right)^2\right) + \rho g (h_1 - h_2) = P_1 + \frac{1}{2} \rho v_1^2 \left(1 - \left(\frac{A_1}{A_2}\right)^2\right) + \rho g (h_1 - h_2)
\]

\[
P_2 = 143 \text{ kPa} + \frac{1}{2} (1 \times 10^3 \text{ kg/m}^3)(1.2 \text{ m/s})^2 (1 - 4) + (1 \times 10^3 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(0 - 0.2 \text{ m})
\]

\[
P_2 = 139 \text{ kPa}
\]
The Bernoulli effect is simple to demonstrate – all you need is a sheet of paper. Hold it by its end, so that it would be horizontal if it were stiff, and blow across the top. The paper will rise, due to the higher speed, and therefore lower pressure, above the sheet.
An airplane wing is shaped so that air flows more rapidly over the top than the bottom. As a result, the pressure on the top is reduced and a net upward force is produced.
15-8 Applications of Bernoulli’s Equation

This lower pressure at high speeds is what rips roofs off houses in hurricanes and tornadoes, and causes the roof of a convertible to expand upward. It even helps prairie dogs with air circulation!
Exercise *Have Wind Insurance?* During a windstorm a 35.5 m/s wind blows over a roof. Find the difference of pressure between the air inside the home and the air just above the roof. $\rho_{\text{air}} = 1.29 \text{ kg/m}^3$
Exercise **Have Wind Insurance?** During a windstorm a 35.5 m/s wind blows over a roof. Find the difference of pressure between the air inside the home and the air just above the roof. $\rho_{\text{air}} = 1.29 \text{ kg/m}^3$

Apply Bernoulli's equation just above the roof and inside the house:

\[ P_1 + \frac{1}{2} \rho v_1^2 + \rho gh_1 = P_2 + \frac{1}{2} \rho v_2^2 + \rho gh_2 \]

\[ P_1 + \frac{1}{2} \rho v_1^2 = P_{\text{atm}} \]

\[ P_{\text{atm}} - P_1 = \frac{1}{2} \rho v_1^2 = \frac{1}{2} (1.29 \text{ kg/m}^3)(35.5 \text{ m/s}) = 813 \text{ Pa} \]
15-8 Applications of Bernoulli’s Equation

**Torricelli’s Law** If a hole is punched in the side of an open container, find the velocity of the fluid emerging from the hole.
15-8 Applications of Bernoulli’s Equation

**Torricelli’s Law** If a hole is punched in the side of an open container, find the velocity of the fluid emerging from the hole.

Apply Bernoulli's equation:

\[ P_1 + \frac{1}{2} \rho v_1^2 + \rho gh_1 = P_2 + \frac{1}{2} \rho v_2^2 + \rho gh_2 \]

\[ P_1 = P_2 = P_{\text{atm}}, \quad v_1 \approx 0, \quad \text{set } h_2 = 0, \quad h_1 = h \]

\[ \rho gh_1 = \frac{1}{2} \rho v_2^2 \Rightarrow v_2 = \sqrt{2gh} \]
If the fluid is directed upwards instead, it will reach the height of the surface level of the fluid in the container.
Example 15-9 A Water Fountain Find D.
Example 15-9 **A Water Fountain** Find D.

Use Torricelli's Law to obtain the speed at which the water emerges from the top tub.

\[ v = \sqrt{2gh} \]

Use the equation of 2D motion to determine D:

\[ x = vt \Rightarrow D = vt \]

\[ y = H - \frac{1}{2}gt^2 \Rightarrow t = \sqrt{\frac{2H}{g}} \]

\[ D = vt = \sqrt{2gh} \sqrt{\frac{2H}{g}} = 2\sqrt{hH} = 0.549\text{m} \]
Viscosity is a form of friction felt by fluids as they flow along surfaces. We have been dealing with nonviscous fluids, but real fluids have some viscosity.

A viscous fluid will have zero velocity next to the walls and maximum velocity in the center.
15-9 Viscosity and Surface Tension

It takes a force to maintain a viscous flow, just as it takes a force to maintain motion in the presence of friction.

A fluid is characterized by its coefficient of viscosity, $\eta$. It is defined so that the pressure difference in the fluid is given by:

$$P_1 - P_2 = 8\pi\eta\frac{vL}{A}$$
15-9 Viscosity and Surface Tension

Using this to calculate the volume flow rate yields:

\[
\text{volume flow rate} = \frac{\Delta V}{\Delta t} = \nu A = \frac{(P_1 - P_2)A^2}{8\pi \eta L} \\
= \frac{(P_1 - P_2)\pi r^4}{8\eta L}
\]

Note the dependence on the fourth power of the radius of the tube!
A molecule in the center of a liquid drop experiences forces in all directions from other molecules. A molecule on the surface, however, experiences a net force toward the drop. This pulls the surface inward so that its area is a minimum.
15-9 Viscosity and Surface Tension

Since there are forces tending to keep the surface area at a minimum, it tends to act somewhat like a spring – the surface acts as though it were elastic.
This means that small, dense objects such as insects and needles can stay on top of water even though they are too dense to float.
Summary of Chapter 15

• Density: \( \rho = \frac{M}{V} \)

• Pressure: \( P = \frac{F}{A} \)

• Atmospheric pressure:

\[
P_{\text{at}} = 1.01 \times 10^5 \text{ N/m}^2 \approx 14.7 \text{ lb/in}^2
\]

• Gauge pressure:

\[
P_g = P - P_{\text{at}}
\]

• Pressure with depth:

\[
P_2 = P_1 + \rho gh
\]
Summary of Chapter 15

• Archimedes’ principle:

   An object completely immersed in a fluid experiences an upward buoyant force equal in magnitude to the weight of fluid displaced by the object.

• Volume of submerged part of object:

   $$V_{\text{sub}} = V_s \left( \frac{\rho_s}{\rho_f} \right)$$
Summary of Chapter 15

- Equation of continuity:
  \[ \rho_1 A_1 v_1 = \rho_2 A_2 v_2 \]

- Bernoulli’s equation:
  \[ P_1 + \frac{1}{2} \rho v_1^2 + \rho g y_1 = P_2 + \frac{1}{2} \rho v_2^2 + \rho g y_2 \]

- Speed of fluid exiting a hole in a container a depth \( h \) below the surface:
  \[ v = \sqrt{2gh} \]
Summary of Chapter 15

• A pressure difference is required to keep a viscous fluid moving:

\[
\frac{\Delta V}{\Delta t} = \frac{(P_1 - P_2) \pi r^4}{8 \eta L}
\]