The Cosmic Perspective

Star Stuff
17.1 Lives in the Balance

- Our goals for learning:
  - How does a star's mass affect nuclear fusion?
How does a star's mass affect nuclear fusion?
Stellar Mass and Fusion

• The **mass** of a main-sequence star **determines** its core pressure and temperature.

• **Stars of higher mass** have higher core temperature and more rapid fusion, making those stars both more luminous and **shorter-lived**.

• **Stars of lower mass** have cooler cores and slower fusion rates, giving them smaller luminosities and **longer lifetimes**.
Stellar Evolution: On and Off the Main Sequence

How long will a star remain on the main sequence? The energy $E$ released over the lifetime $t$ of a star is:

$$E = fMc^2,$$

where $f$ is the fraction of the star's mass that's converted into energy

$$L = \frac{E}{t} \Rightarrow t = \frac{E}{L} = \frac{fMc^2}{L} \propto \frac{M}{M^{3.5}} = \frac{1}{M^{2.5}}$$

$$t_{\text{star}} = t_{\text{solar}} \left(\frac{M_{\text{solar}}}{M_{\text{star}}}\right)^{2.5}, \quad t_{\text{solar}} = 1.2 \times 10^{10} \text{ years}$$

<table>
<thead>
<tr>
<th>Mass ($M_\odot$)</th>
<th>Surface temperature (K)</th>
<th>Spectral class</th>
<th>Luminosity ($L_\odot$)</th>
<th>Main-sequence lifetime ($10^6$ years)</th>
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<tbody>
<tr>
<td>2.5</td>
<td>35,000</td>
<td>O</td>
<td>80,000</td>
<td>4</td>
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<tr>
<td>1.5</td>
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<td>4000</td>
<td>M</td>
<td>0.03</td>
<td>700,000</td>
</tr>
</tbody>
</table>

The main-sequence lifetimes were estimated using the relationship $t \propto 1/M^{2.5}$ (see Box 19-2).
High-Mass Stars
$> 8M_{\text{Sun}}$

Intermediate-Mass Stars

Low-Mass Stars
$< 4M_{\text{Sun}}$

Red Dwarfs
$0.08 \ M_{\text{Sun}} < M < 0.4 \ M_{\text{Sun}}$

Brown Dwarfs
Stellar Evolution: On and Off the Main Sequence

Case of $0.08 \, M_\odot < M_{\text{star}} < 0.4 \, M_\odot$

These stars are called **red dwarfs** because they are less massive than the sun and they are red in color due to their low temperature.

85% of the stars in the Milky Way are M class red dwarfs. **Energy in red dwarfs is transported by convection** from the core to the outer layers and therefore they do not build up He only in the core as do stars with $> 0.4 \, M_\odot$. 
Case: $0.08 \, M_\odot < M_{\text{star}} < 0.4 \, M_\odot$

**Question:** What happens when the H runs out?

**Answer:** In a red dwarf when the H runs out the temperature and density are not high enough to initiate fusion of He.

After H fusion ends in a red dwarf it radiates its energy away, slowly cools and shrinks in size.

Calculations indicate that it takes ~ 100 billion years for a red dwarf to convert all of its H to He.
Low Mass Stars: $4 \, M_\odot > M_{\text{star}} > 0.4 \, M_\odot$

**Question:** What happens when the H runs out?

**Answer:** When H runs out in the core **H fusion continues in a shell around the core.**

The core cannot support the material above it so it **begins to shrink** and is compressed from the weight of the outer layers. When the gas in the core becomes compressed its temperature again begins to rise.
Case of $4 \, M_\odot > M_{\text{star}} > 0.4 \, M_\odot$

**Answer continued:** The increase in core temperature increases the rate of H fusion reactions in the thin H shell that surrounds the shell while there is no core H fusion.

The outer layers expand by the increased luminosity of the H burning shell. As the external layers of gas expand the surface temperature of the star decreases.

When the temperature of the external layers drops to $\sim 3,500$ K the star appears reddish at which point is has become a **red giant**.
Case of $4 \, M_\odot > M_{\text{star}} > 0.4 \, M_\odot$

A star leaves the main sequence when hydrogen fusion ends in the core. The star becomes a red giant.
When the sun becomes a red giant its diameter will increase from \(~0.01\) AU to \(~2\) AU.
Red giants lose a substantial amount of gas from the outer layers.

The reason for this is that the outer layers have expanded far from the center and since force is proportional to $1/R^2$ the force keeping the gas bound is substantially weaker.

**Mass loss from red giants** is detected from blue-shifted emission lines that imply wind velocities of 10 km/s. A red giant may lose $\sim 10^{-7} \, M_\odot$ per year (the sun loses $\sim 10^{-14} \, M_\odot$ per year).
While still on the main sequence the Sun’s luminosity will continue to increase. In about 3.5 billion years the average temperature on Earth will be $> 100 ^\circ C$ and the water on the surface will boil away.

**H** fusion reactions in the core are expected to cease in about 7.5 billion years from now!

At about $\sim 7.7$ billion years from now the Sun will have expanded to a diameter of about 2 AU.
Stellar Evolution: On and Off the Main Sequence

Outer layers: no thermonuclear reactions

Hydrogen-fusing shell

Hydrogen-fusing core
Main-sequence star

Helium core, no thermonuclear reactions
Young red-giant star

Helium-fusing shell
Helium-fusing core
Red-giant star after helium fusion begins
Thought Question

What happens when a star can no longer fuse hydrogen to helium in its core?

A. The core cools off.
B. The core shrinks and heats up.
C. The core expands and heats up.
D. Helium fusion immediately begins.
Thought Question

What happens when a star can no longer fuse hydrogen to helium in its core?

A. The core cools off.
B. The core shrinks and heats up.
C. The core expands and heats up.
D. Helium fusion immediately begins.
• Helium fusion does not begin right away because it requires higher temperatures than hydrogen fusion—larger charge leads to greater repulsion.

• Fusion of two helium nuclei doesn't work, so helium fusion must combine three helium nuclei to make carbon.
Core Helium Fusion: The Triple Alpha Process

1. In the helium-rich core of a red giant, temperatures and pressures become high enough for helium nuclei ($^4\text{He}$) to collide and fuse together.

2. Fusing two $^4\text{He}$ nuclei produces a beryllium nucleus ($^7\text{Be}$) with four protons and four neutrons. The $^7\text{Be}$ nucleus is very unstable: it breaks apart, leaving two $^4\text{He}$ nuclei again, with a half-life of just $7 \times 10^{-17}$ s.

3. However, if a third $^4\text{He}$ nucleus collides with the $^7\text{Be}$ nucleus during its brief existence, the result is a stable nucleus of carbon ($^{12}\text{C}$) with six protons and six neutrons.

4. This nuclear reaction releases a gamma-ray photon ($\gamma$). The photon's energy goes into sustaining the internal heat of the red giant.

5. Some $^{12}\text{C}$ nuclei collide with an additional $^4\text{He}$ nucleus to form a stable nucleus of oxygen ($^{16}\text{O}$) with eight protons and eight neutrons.

6. This reaction, too, releases a gamma-ray photon whose energy helps sustain the red giant's internal heat.
Core He Fusion

The way He fusion begins in the core of a star depends on its mass.

for $0.4 \, M_\odot < M_{\text{star}} < 2-3 \, M_\odot$ Helium flash

$M_{\text{star}} > 2-3 \, M_\odot$ we have gradual ignition of He
Safety Valve

In most cases the gases of a star obey the law: $VT^{\gamma -1} = \text{constant}$, where $V$=volume, $T$=temperature and $\gamma$ is the adiabatic index that depends on the type of gas ($\gamma = 1.4$ for diatomic gas). **When the gas expands the temperature goes down.**

When the rate of fusion reactions increases, the outward radiation pressure also increases and expands the core. This expansion results in a decrease in temperature and the reaction rate goes down. This provides a safety valve for the star.

Conversely, a decrease in the reaction rate causes a compression of the gas resulting in an increase of the core temperature and the reaction rate goes up.
Closely packed electrons resist compression. The reason for this is that no two electrons too close to each other can have the same four quantum numbers \( n, l, m_l, m_s \) (The Pauli exclusion principle).

The pressure of the electrons resisting compression is called degeneracy pressure.

In a red giant with \( 0.4 \, M_\odot < M_{\text{star}} < 2-3M_\odot \) the core must be compressed tremendously in order to become hot enough for helium fusion to begin.

Electron degeneracy pressure prevents a low mass red giant from compressing any further.
Helium flash is the sudden beginning of helium fusion in the core of intermediate mass stars of less than about 2.25 solar masses. The core is held up mostly by degeneracy pressure.

The cores of $0.4 \, M_{\text{solar}} < M_{\text{star}} < 2-3 \, M_{\text{solar}}$ red giant stars are degenerate. A helium flash occurs when fusion begins in a degenerate gas.

The cores of $M_{\text{star}} > 2-3 \, M_{\text{solar}}$ red giant stars are not degenerate. In these stars He fusion begins gradually with no flash.
Thought Question

What happens in a low-mass star \((0.4 \, M_{\text{sol}} < M_{\text{star}} < 2-3 \, M_{\text{sol}})\) when the core temperature rises enough for helium fusion to begin?

A. Helium fusion slowly starts.
B. Hydrogen fusion stops.
C. Helium fusion rises very sharply.

*Hint: Degeneracy pressure is the main form of pressure in the inert helium core.*
Thought Question

What happens in a low-mass star when the core temperature rises enough for helium fusion to begin?

A. Helium fusion slowly starts.
B. Hydrogen fusion stops.
C. Helium fusion rises very sharply.

Hint: Degeneracy pressure is the main form of pressure in the inert helium core.
Life Track after Helium Flash

• Models show that a red giant should shrink and become less luminous after helium fusion begins in the core.

• During helium fusion in the core the star lies on the horizontal branch.

Helium fusion begins with the helium flash, after which the star’s surface shrinks and heats, making the star’s life track move downward and to the left on the H-R diagram.
1. The star shines by shell hydrogen fusion: The inert core shrinks and the outer layers expand.

2. Luminosity increases and surface temperature decreases, so the star moves up and to the right on the H-R diagram (along the red-giant branch).

3. Core helium fusion begins with the helium flash (*).

4. The star now shines by shell hydrogen fusion and core helium fusion: The core expands and the outer layers shrink.

5. Luminosity decreases and surface temperature increases, so the star moves down and to the left on the H-R diagram (into the horizontal branch).

6. Eventually all of the core helium is used up.

7. The star now shines by shell hydrogen fusion and shell helium fusion: The core shrinks and the outer layers expand.

8. Luminosity increases and surface temperature decreases, so the star moves up and to the right on the H-R diagram (along the asymptotic giant branch).

9. Eventually the star sheds its outer layers to form a planetary nebula.

(a) Before the helium flash: A red-giant star

(b) After the helium flash: A horizontal-branch star

(c) After core helium fusion ends: An AGB star
The Death of Low Mass Stars

He fusion produces O and C in the core. In a 1 $M_\odot$ star He fusion will last for about $10^8$ years.

When He runs out, the core will collapse again resulting in an increase in the core temperature ($T$). This increase in $T$ results in an increase in the reaction rate in the He shell that leads to the expansion of the outer layers for a second time. The resulting star is called an asymptotic giant branch (AGB) star.
The Death of Low Mass Stars

The convection zone can grow significantly during the AGB phase of a $M < 4M_\odot$ star.

This broadening of the convection zone *drudges-up heavier elements* (ie. C, N and O) from the core to the surface.

The *drudged-up carbon* produces strong absorption lines in the spectra of some AGB stars. These are called *carbon stars*.

Carbon AGB star TT Cygni showing radio emission from CO in the ejected shell. AGB stars have very powerful winds with outflow rates of $10^{-4} M_\odot$ per year. A red giant’s outflow is $\sim 10^{-7} M_\odot$ per year and the sun’s is $\sim 10^{-14} M_\odot$ per year.
How does a low-mass star die?

The Helix Nebula, is a large planetary nebula. The estimated distance is about 700 light-years. Its age is estimated to be about 10,000 years.

This observation of the Helix Nebula was made with the College of Charleston 24 inch CDK PlaneWave telescope.

Observers: CofC Students: Lucy Williamson and Dereck Morgado, CofC Faculty: Dr. Ashley Pagnotta and Dr. George Chartas.
Thought Question

What happens when the star's core runs out of helium?

A. The star explodes.
B. Carbon fusion begins.
C. The core cools off.
D. Helium fuses in a shell around the core.
Thought Question

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Planetary Nebulae

• Double shell burning ends with a pulse that ejects the H and He into space as a *planetary nebula*.

• The core left behind becomes a white dwarf.
One unusual property of white dwarfs is the larger the mass the smaller the radius.

There is a limit however as to how much mass a white dwarf can have. Above this limit degeneracy pressure cannot overcome gravity. The limit is $1.4 \, M_\odot$ and was first derived by Indian astronomer Chandrasekhar.

When a white dwarf is initially formed it consists of ionized carbon and oxygen atoms floating in a sea of degenerate electrons. As the white dwarf cools the motion of the C and O reduces to the point where the atoms form a crystal lattice.

A cool carbon-oxygen white dwarf resembles a huge diamond.

The mass-radius relation for a white dwarf. The maximum mass of a white dwarf, called the Chandrasekhar limit, is $1.4 \, M_\odot$. 
White Dwarfs

During the asymptotic giant branch a star ejects its outer layers exposing the burned-out core.

Once all the outer layers are ejected and the planetary nebula fades away the core continues to cool down and its luminosity decreases. **A white dwarf’s size does not change because it is supported by degeneracy pressure.**

The fraction of material ejected will depend on the mass of the giant star.

<table>
<thead>
<tr>
<th>Evolutionary track</th>
<th>Giant star</th>
<th>Ejected nebula</th>
<th>White dwarf</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
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<td>1.2</td>
</tr>
<tr>
<td>B</td>
<td>1.5</td>
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</tr>
<tr>
<td>C</td>
<td>0.8</td>
<td>0.2</td>
<td>0.6</td>
</tr>
</tbody>
</table>

Evolution from Giants to White dwarf
End of Fusion

• Fusion progresses no further in a low-mass star because the core temperature never grows hot enough for fusion of heavier elements (some helium fuses to carbon to make oxygen).

• Degeneracy pressure supports the white dwarf against gravity.
Life Track of a Sun-like Star
Earth's Fate

- The Sun's luminosity will rise to 1000 times its current level—too hot for life on Earth.
Earth's Fate

The Sun's radius will grow to near current radius of Earth's orbit.
What have we learned?

• **What are the life stages of a low-mass star?**
  – Hydrogen fusion in core (main sequence)
  – Hydrogen fusion in shell around contracting core (red giant)
  – Helium fusion in core (horizontal branch)
  – Double shell burning (AGB red giant)

• **How does a low-mass star die?**
  – Ejection of hydrogen and helium in a planetary nebula leaves behind an inert white dwarf.
17.3 Life as a High-Mass Star

• Our goals for learning:
  – What are the life stages of a high-mass star?
  – How do high-mass stars make the elements necessary for life?
  – How does a high-mass star die?
CNO Cycle

- High-mass main-sequence stars fuse H to He at a higher rate using carbon, nitrogen, and oxygen as catalysts.

- Greater core temperature enables hydrogen nuclei to overcome greater repulsion.
Life Stages of High-Mass Stars

• Late life stages of high-mass stars are similar to those of low-mass stars:
  – Hydrogen core fusion (main sequence)
  – Hydrogen shell burning (supergiant)
  – Helium core fusion (horizontal branch)
Life Stages of High-Mass Stars

The reason why high densities and temperatures are required to fuse elements heavier than He is that heavy nuclei have large charges and therefore large electric forces that tend to keep the nuclei apart.

When a star with initial mass $M > 4M_\odot$ runs out of He in its core the core begins to contract and the outer layers of the star expand.

Because the mass of the core of a star with $M > 4M_\odot$ is more than the Chandrasekhar limit of 1.4 $M_\odot$, degeneracy pressure cannot prevent the collapse of the star. The temperature continues to rise and when it reaches 600 million K carbon fusion begins.
Carbon fusion: $T_{\text{Fusion}} = 600 \times 10^6 \text{ K}$
Products = oxygen (O), neon (Ne), sodium (Na), magnesium (Mg)

If the star has $M > 8M_\odot$ additional reactions can occur:
Neon fusion: $T_{\text{Fusion}} = 10^9 \text{ K}$
Products = oxygen (O), magnesium (Mg)

Oxygen fusion: $T_{\text{Fusion}} = 1.5 \times 10^9 \text{ K}$
Product = silicon (Si), magnesium (Mg), phosphorus (P), sulfur(S)

Silicon fusion: $T_{\text{Fusion}} = 2.7 \times 10^9 \text{ K}$
Products = Sulfur (S), Iron (Fe), Nickel (Ni)
• Advanced nuclear burning proceeds in a series of nested shells.
• Iron is a dead end for fusion because nuclear reactions involving iron do not release energy.

• (This is because iron has lowest mass per nuclear particle.)
How does a high-mass star die?
Core Collapse

A massive star with $M > 8M_{\odot}$ will eventually reach the point of fusing Si into Fe. **Fusion of Fe does not produce energy so the core cannot generate heat from fusion but instead begins to collapse and heat up.**

• The gamma rays emitted from the hot core have very large energies that break the iron into helium nuclei and free neutrons. This process is called **photodisintegration**.

• Because of the high density, electrons combine with protons to form neutrons and neutrinos: $e^- + p^+ \rightarrow n + \nu$

• The escape of the neutrinos ($\nu$) **cools down the core** and leads to more compression. Seconds after the contraction began the density reaches the nuclear density value of $\sim 4 \times 10^{17} \text{kg m}^{-3}$. (compare with the density of water $\sim 1000 \text{ kg m}^{-3}$ )
Core Collapse

• **Electron degeneracy pressure** goes away because electrons combine with protons, making neutrons and neutrinos.

• Neutrons collapse to the center, forming a **neutron star**.
Core Bounce

The core becomes very rigid at nuclear density and further compression results in the innermost part of the core **bouncing back** and sending out a pressure wave.

There isn’t enough radiation pressure to hold the outer layers up against gravity, and the material from the outer layers plunges inward at speeds of up to 20% of the speed of light.

When the infalling material encounters the outward moving pressure wave it changes direction and starts moving outward reaching supersonic speeds. The **outflowing neutrinos** are thought to **contribute to the outward shock wave**.

**It takes a few hours for the shock wave to reach the surface** and lift away the outer layers of the star.
Supernova Explosions

1. As the massive star nears its end, it takes on an onion-layer structure. At this point in its evolution the star is hundreds of millions of kilometers in radius; only its inner regions are shown here.

2. Iron does not undergo nuclear fusion, so the core becomes unable to generate heat. The gas pressure drops, and overlying material suddenly rushes in.

3. Within a second, the core collapses to nuclear density. Inward-falling material rebounds off the core, setting up an outward-going pressure wave.

4. Neutrinos pouring out of the nascent neutron star propel the shock wave outward, unevenly.

5. The shock wave sweeps through the entire star, blowing it apart.
Energetics

The energy released in a core-collapse supernova explosion is extremely large at ~ $10^{44}$ Joules! (energy emitted in radiation).

The energy released in the form of neutrinos is about 100 times larger than the energy released in radiation.

The total energy released in a core-collapse supernova explosion comes from gravitational energy released by the collapse of the core and the infall of the outer layers of the star.
Material ejected from a massive star during a core-collapse supernova is compressed by the outward shock wave and thermonuclear reactions occur in the gas that produce elements, such as oxygen (O), Magnesium (Mg), Sodium (Na), and iron (Fe).

The following heavy elements can also be produced by merging neutron stars: silver (Ag), tin (Sn), gold (Au), mercury (Hg), lead (Pb), and uranium (U). Gallium (Ga), Germanium (Ge), Arsenic (As), Krypton (Kr).
Element Origins

- Merging Neutron Stars
- Dying Low Mass Stars
- Exploding Massive Stars
- Exploding White Dwarfs
- Big Bang
- Cosmic Ray Fission

Based on graphic created by Jennifer Johnson
Supernova Remnant

- Energy released by the collapse of the core drives the star's outer layers into space.

- The Crab Nebula is the remnant of the supernova seen in A.D. 1054 (6500ly away).
The closest supernova in the last four centuries was seen in 1987 (~170,000 ly away).
Supernova 1987A

Light from SN 1987 A continues to arrive. It doesn't all come out at one short period.

SN 1987 A is also emitting gamma rays from the decay of radioactive isotopes created during the supernova explosion.

Five very bright supernovae within our Galaxy have been observable without a telescope in the last 1000 years.

On Feb 1987 a supernova was discovered in the Large Magellanic Cloud. It was so bright it could be seen without a telescope in the southern hemisphere.
Rings around Supernova 1987A

About 20,000 years before the explosion a shell of gas was ejected from the red giant and about 10,000 years before the explosion another shell of gas was ejected.

The UV flash of the supernova explosion ionized the ejected stellar material and caused it to glow.
Rate of Supernovae in the Milky Way

From the frequency with which supernovae occur in distant galaxies, it is reasonable to suppose that a galaxy such as our own should have as many as five supernovae per century.

Where have they been?

A composite of observations of SNR Cassiopeia A at X-ray, visible, and infrared wavelengths.

Cas-A is about 11,000 ly away and the photons from the supernova explosion arrived ~300 years ago.
What have we learned?

• What are the life stages of a high-mass star?
  – They are similar to the life stages of a low-mass star.

• How do high-mass stars make the elements necessary for life?
  – Higher masses produce higher core temperatures that enable fusion of heavier elements.

• How does a high-mass star die?
  – Its iron core collapses, leading to a supernova.
17.4 The Roles of Mass and Mass Exchange

• Our goals for learning:
  – How does a star's mass determine its life story?
  – How are the lives of stars with close companions different?
Role of Mass

• A star's mass determines its entire life story because it determines its core temperature.

• High-mass stars with $> 8M_{\text{Sun}}$ have short lives, eventually becoming hot enough to make iron, and end in supernova explosions.

• Low-mass stars with $< 4M_{\text{Sun}}$ have long lives, never become hot enough to fuse carbon nuclei, and end as white dwarfs.

• Intermediate-mass stars can make elements heavier than carbon but end as white dwarfs.
Low-Mass Star Summary

1. Main sequence: H fuses to He in core.
2. Red giant: H fuses to He in shell around He core.
3. Helium core burning: He fuses to C in core while H fuses to He in shell.
4. Double shell burning in AGB: H and He both fuse in shells.
5. Planetary nebula leaves white dwarf behind.
1. Main sequence: H fuses to He in core.
2. Red supergiant: H fuses to He in shell around He core.
3. Helium core burning: He fuses to C in core while H fuses to He in shell.
4. Multiple shell burning: Many elements fuse in shells.
5. Supernova leaves neutron star or black hole behind.
How are the lives of stars with close companions different?

Algol at onset of mass transfer. When the more massive star expanded into a red giant, it began losing some of its mass to its normal, hydrogen core fusion companion.
Thought Question

• The binary star Algol consists of a $3.7M_{\text{Sun}}$ main-sequence star and a $0.8M_{\text{Sun}}$ subgiant star.

• What's strange about this pairing?

• How did it come about?
• The stars in Algol are close enough that matter can flow from the subgiant onto the main-sequence star.
• The star that is now a subgiant was originally more massive.

• As it reached the end of its life and started to grow, it began to transfer mass to its companion (mass exchange).

• Now the companion star is more massive.
What have we learned?

• How does a star's mass determine its life story?
  – Mass determines how high a star's core temperature can rise and therefore determines how quickly a star uses its fuel and what kinds of elements it can make.

• How are the lives of stars with close companions different?
  – Stars with close companions can exchange mass, altering the usual life stories of stars.
Helium Capture

- High core temperatures allow helium to fuse with heavier elements.

**a** Helium-capture reactions.
• Core temperatures in stars with $>8M_{\text{Sun}}$ allow fusion of elements as heavy as iron.