

Postcards From The Edge and Gravitational Waves

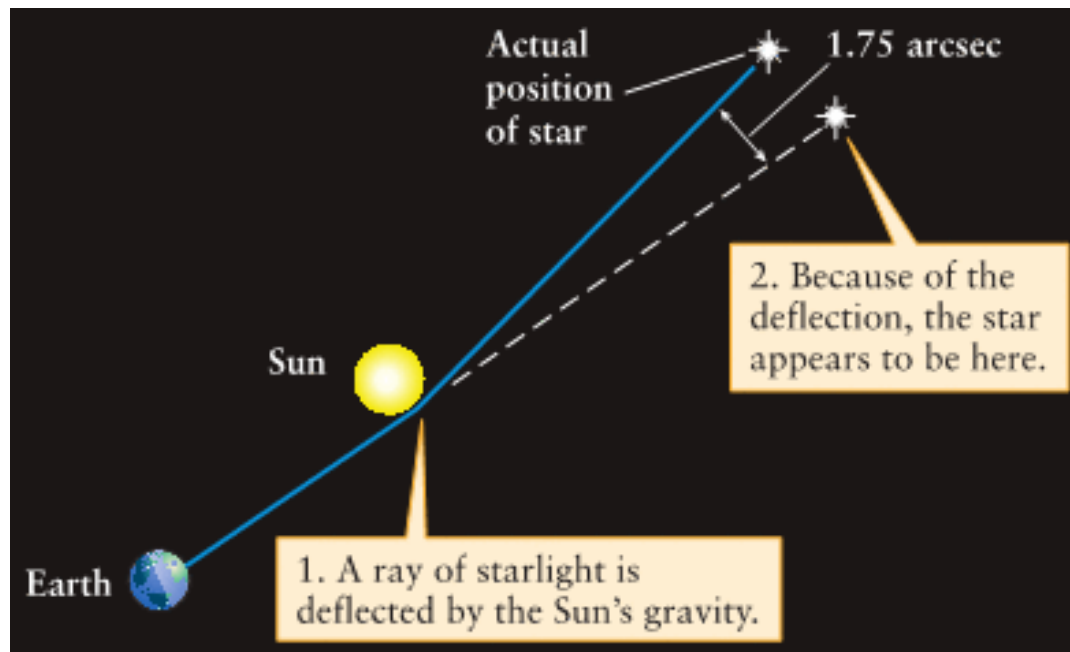


Checking Up on Einstein

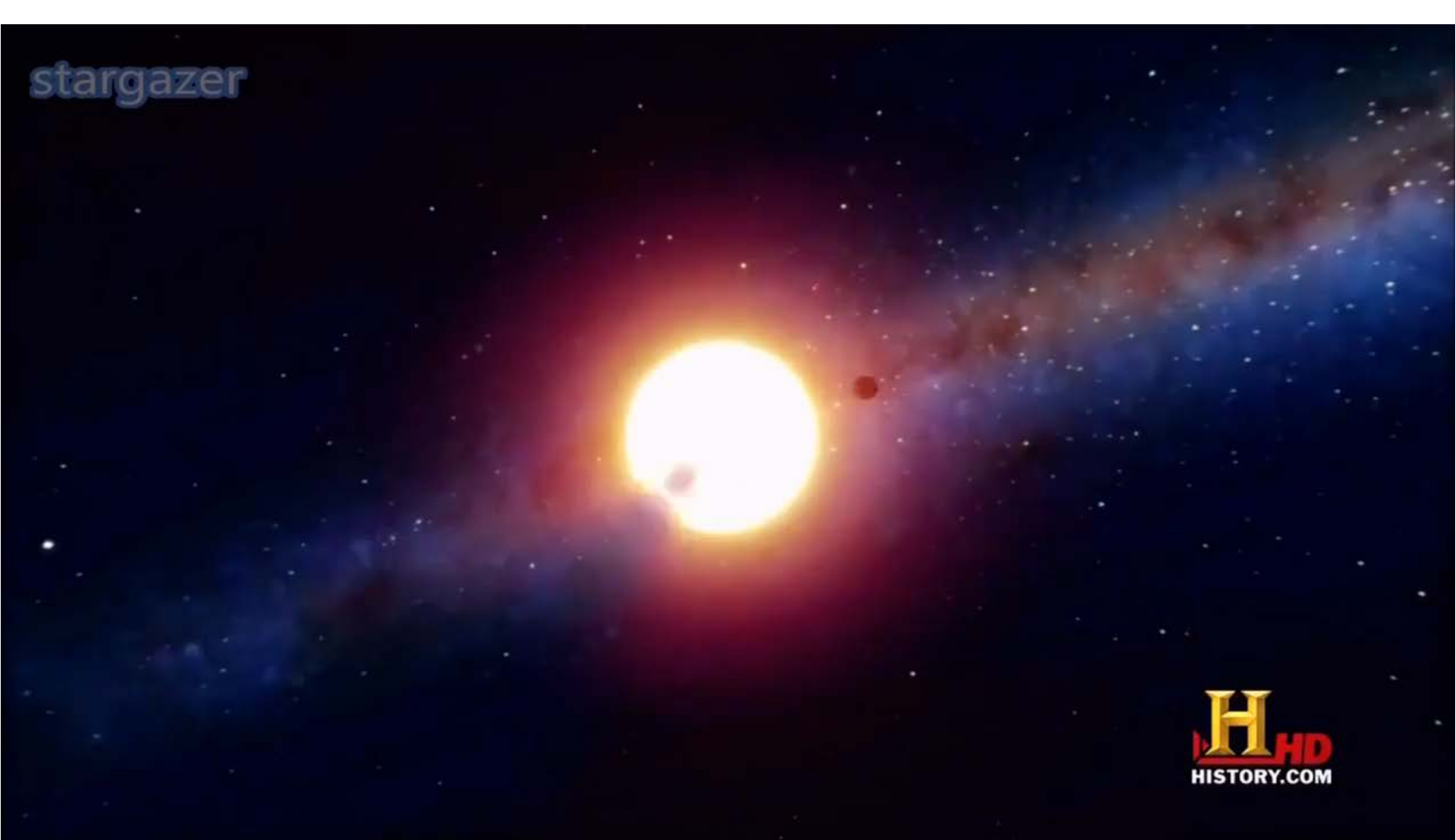
The first test of Einstein's theory was the **bending of light** as it passes a massive object. This effect was conclusively observed by Arthur Eddington during the solar eclipse of 1919.

$$\tilde{\alpha} = 4GM/r_0c^2 \text{ (in units of radians)}$$

$$M_{\odot} = 2 \times 10^{30}\text{kg}, r_{\odot} = 695.7 \times 10^6 \text{ m}, G = 6.67408 \times 10^{-11}$$



stargazer



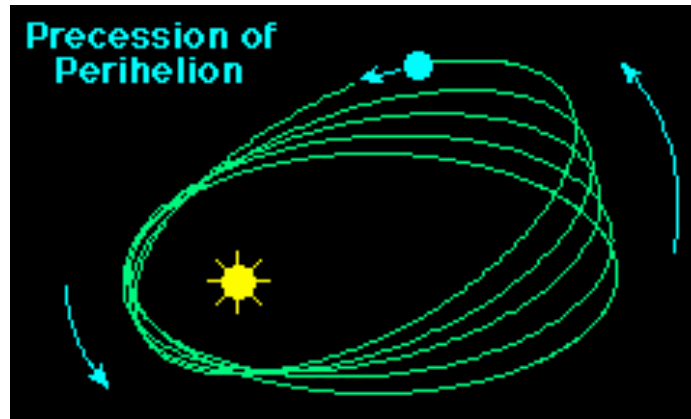
H **HD**
HISTORY.COM

<https://www.youtube.com/watch?v=8xwGE1oUoSU>

[**https://www.youtube.com/watch?v=5Z6EItMIU7k**](https://www.youtube.com/watch?v=5Z6EItMIU7k)

Checking Up on Einstein

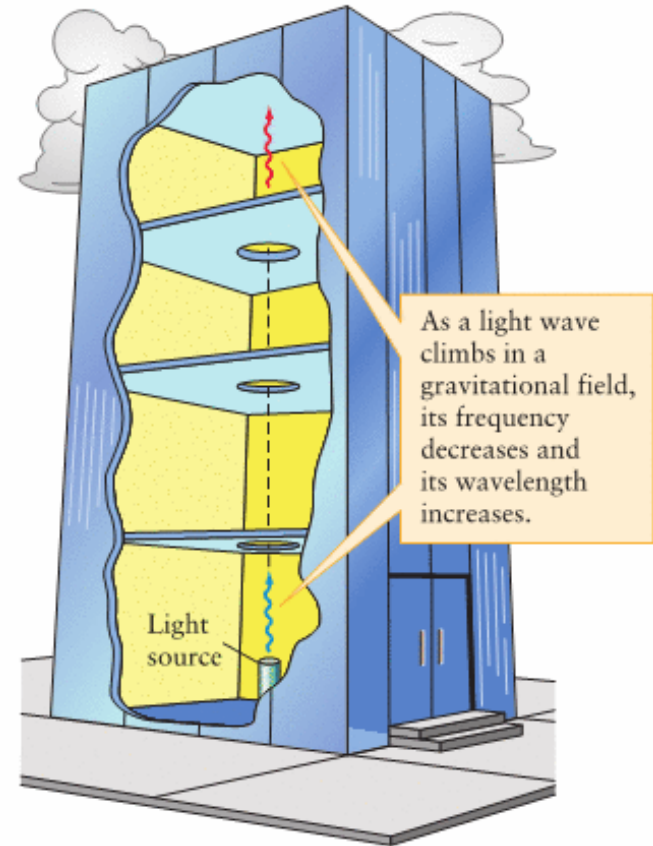
In a second test Einstein's theory accurately explained the **precession of Mercury's orbit** around the sun. Most of the precession can be accounted for by Newtonian physics but an additional amount of about 43 arcsec per century could not be explained by Newtonian physics.



Checking Up on Einstein

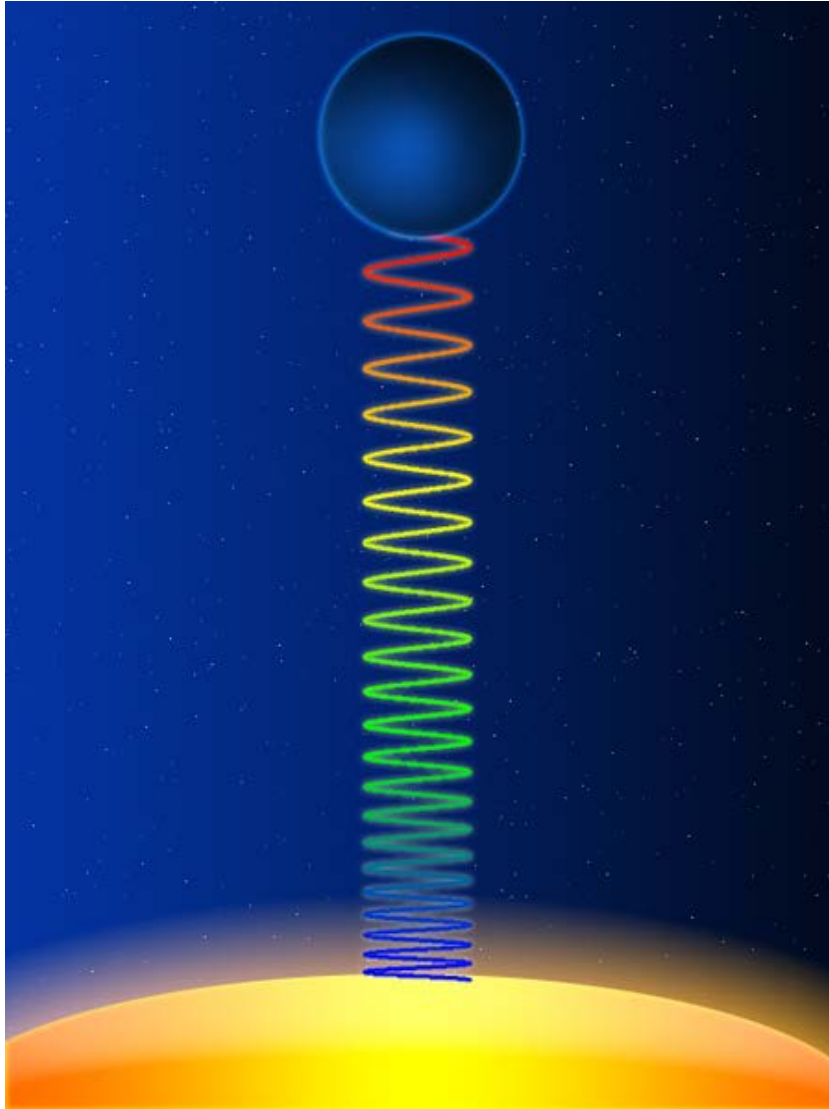
The General Theory of Relativity predicts that the wavelength of light is redshifted by gravity.

In 1960, Robert V. Pound and Glen A. Rebka demonstrated that a beam of very high energy **gamma rays** was ever so slightly **redshifted as it climbed out of Earth's gravity** and up an elevator shaft in the Jefferson Tower physics building at Harvard University.



(b) The gravitational redshift

Checking Up on Einstein



$$\frac{\nu_{obs}}{\nu_{em}} = \frac{\sqrt{\left(1 - \frac{2GM}{c^2 r_{em}}\right)}}{\sqrt{\left(1 - \frac{2GM}{c^2 r_{obs}}\right)}}$$

Checking Up on Einstein

Testing General Relativity near Black Holes.

In 1963 **Kerr** provided an exact solution of Einstein's equations for all **spinning black holes** in the Universe.

How accurate is Kerr's solution of Einstein's equations of BHs?

What observations can one perform to test Einstein's theory of general relativity in the strong gravity regime?

Are all black holes of the same spin and mass alike as predicted by Kerr's solution?

Checking Up on Einstein

Tests:

Observe the distortion of space and time near a black hole and compare it to the predicted value of Kerr's solution.



Far away from the event horizon the effects of gravity on accreting gas can be approximately described with Newton's theory. Relativistic effects however would become detectable near the event horizon:

According to Kerr's solution the inner radius of the accretion disk depends on the spin of the black hole. The faster the spin the smaller the radius of the innermost stable orbit and the larger amount of energy radiated.

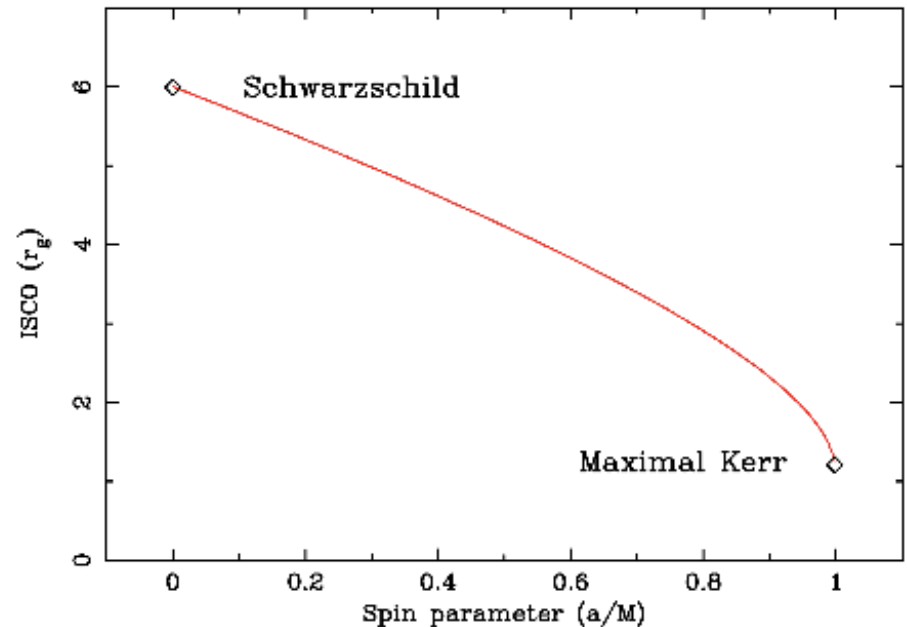
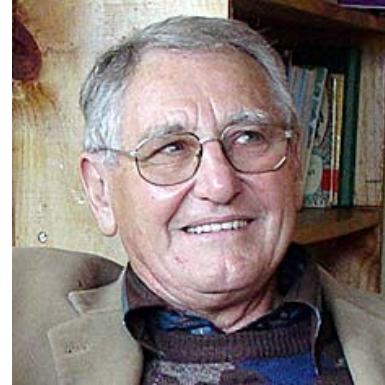
Checking Up on Einstein

Innermost Stable Circular Orbit (ISCO)

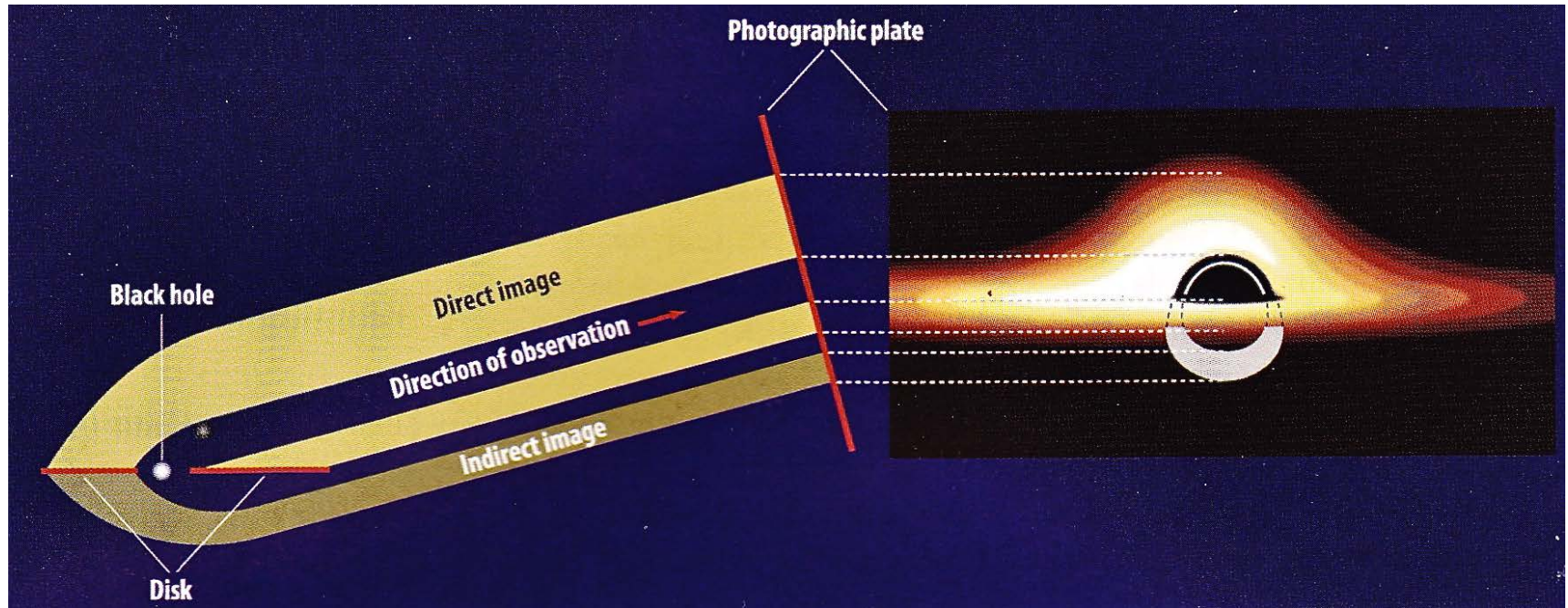
$$r_{\text{ISCO}} (\text{no spin}) = 6GM_{\text{BH}}/c^2 = 6r_g = 3R_s$$

$$r_{\text{ISCO}} (\text{max spin}) = r_g$$

A particle may be in a circular orbit with $r < r_{\text{ISCO}}$ but it will not be stable and eventually it will fall into the black hole.



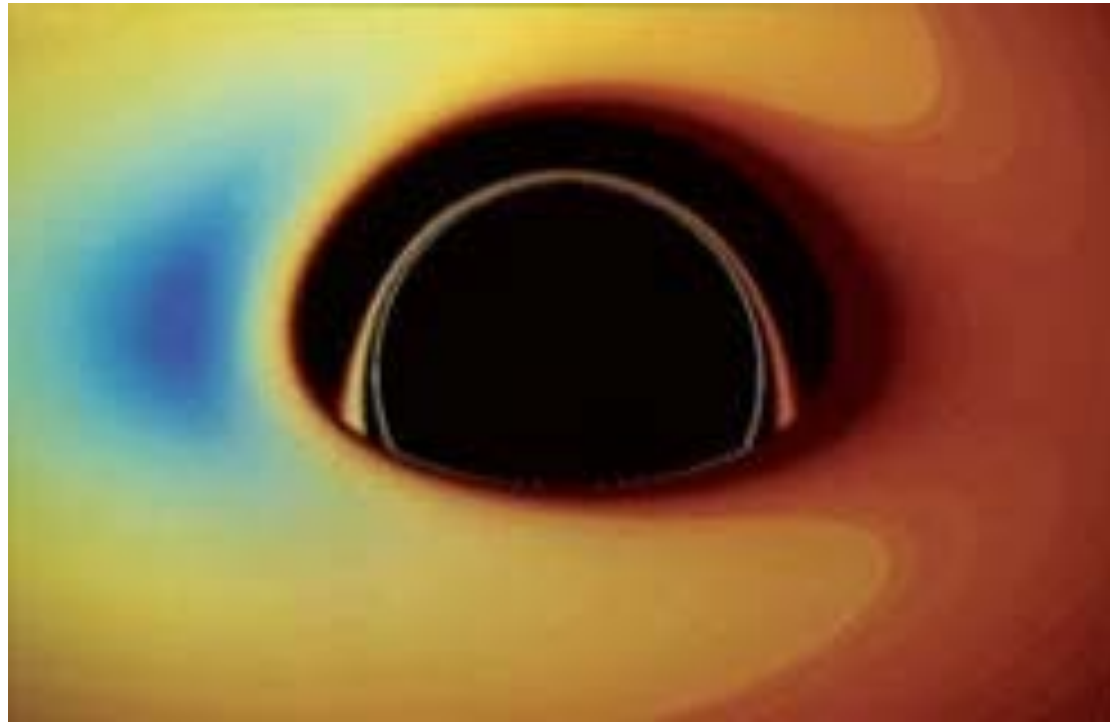
Predicted Distortion of a BH Accretion Disk



A thin disk viewed at an angle of 10° to its plane (inclination angle of 80°).

Rays from the far side are bent over and under the BH. Doppler beaming associated with the fast rotation of the disk make the approaching side more luminous than the receding side.

Predicted Distortion of a BH Accretion Disk



A simulation of a thin disk viewed at an angle of 30° to its plane. Doppler beaming associated with the fast rotation of the disk make the left approaching side more luminous than the receding side. The narrow arc over the BH is due to rays from the underside of the foreground disk that are bent by almost 180° and travel through gap between the horizon and accretion disk. The ring is produced by photons that orbit the BH once before escaping towards our direction.

Predicted Distortion of a BH Accretion Disk

The black hole will cast a shadow of about $5 R_s$ against the background glow.

What is the expected **angular size** of this shadow for :

(a) Sgr A*, $M_{\text{BH}} = 4 \times 10^6 M_{\odot}$, $D = 8 \text{ kpc}$

(b) M87, $M_{\text{BH}} = 6.4 \times 10^9 M_{\odot}$, $z = 0.004360$ (0.089 kpc per arcsec)

(c) M31, $M_{\text{BH}} = 4 \times 10^7 M_{\odot}$, $D = 778 \text{ kpc}$

1 kpc $\sim 3.09 \times 10^{19}$ meters

$$\theta = \frac{l}{2\pi D} 360^\circ$$

Where D is the distance to the object, $l \sim 5R_s$ is the linear size of the shadow and θ is the angular size of the shadow.

Predicted Distortion of a BH Accretion Disk

(a) SgrA*

$$R_{shadow} = 5R_s = 5. \times 3. \times 4. \times 10^6 \text{ km} = 6 \times 10^7 \text{ km}$$

$$\theta = \frac{R_{shadow}}{2\pi D} 360^\circ = \frac{6 \times 10^7 \text{ km}}{2\pi \times 8 \text{ kpc}} 360^\circ = \frac{6 \times 10^7 \text{ km}}{16\pi \times 3.086 \times 10^{16} \text{ km}} 360^\circ = 50 \mu\text{arcsec}$$

(b) M87

$$R_{shadow} = 5R_s = 5. \times 3. \times 6.4 \times 10^9 \text{ km} = 9.6 \times 10^{10} \text{ km}$$

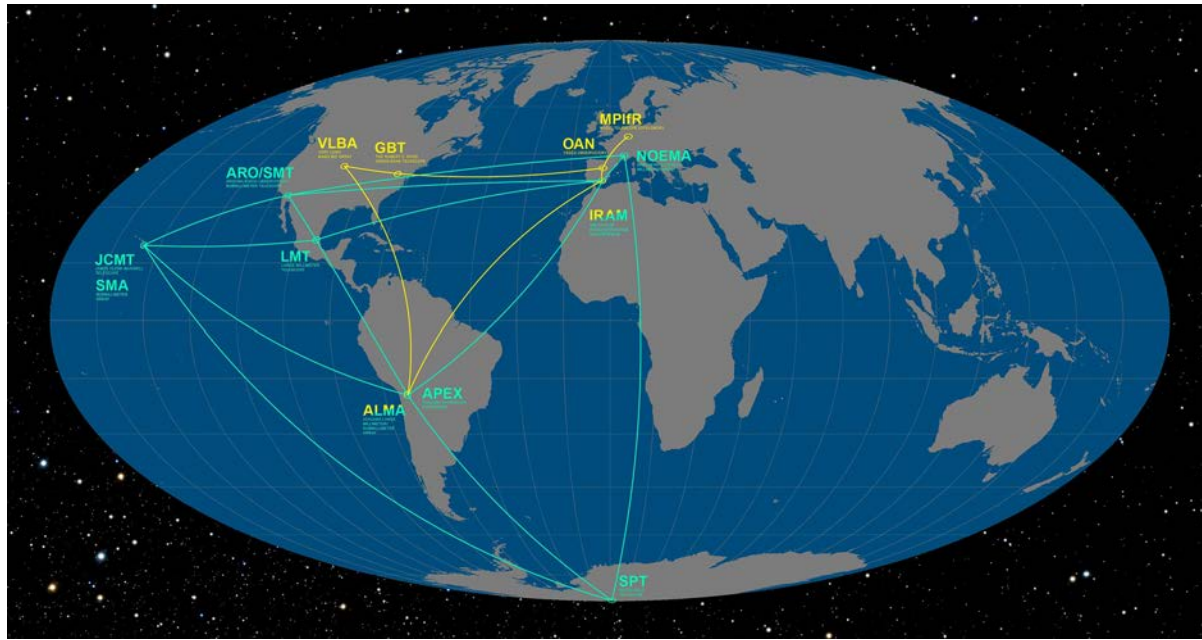
$$0.089 \times 3.086 \times 10^{16} \text{ km per arcsec} \Rightarrow \theta = \frac{9.6 \times 10^{10} \text{ km}}{0.089 \times 3.086 \times 10^{16} \text{ km}} \text{ arcsec} = 35 \mu\text{arcsec}$$

(c) M31

$$R_{shadow} = 5R_s = 5. \times 3. \times 4. \times 10^7 \text{ km} = 6 \times 10^8 \text{ km}$$

$$\theta = \frac{R_{shadow}}{2\pi D} 360^\circ = \frac{6 \times 10^8 \text{ km}}{2\pi \times 778 \text{ kpc}} 360^\circ = \frac{6 \times 10^8 \text{ km}}{1556\pi \times 3.086 \times 10^{16} \text{ km}} 360^\circ = 5.2 \mu\text{arcsec}$$

Imaging a Black Hole



The Event Horizon Telescope (EHT) is a large telescope array consisting of a global network of radio telescopes operating at short wavelengths \sim mm .

The EHT project combines data from several very-long-baseline interferometry (VLBI) stations around Earth, which form a combined array with an angular resolution **sufficient to resolve the supermassive black hole shadows of M87 and SgrA***.

Diffraction

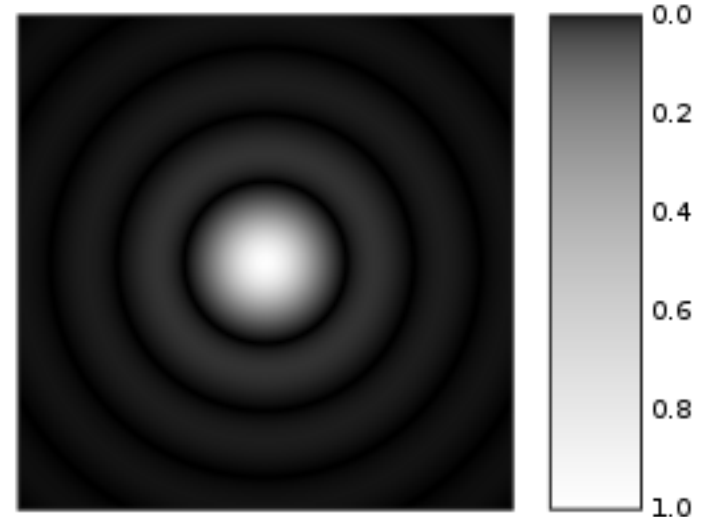
The light collected by a telescope at the focal point is not focused to a point but forms a **diffraction pattern** having a central peak with an angular size between the peak and the first null:

$$\theta = 2.5 \times 10^5 \frac{\lambda}{D}$$

θ = diffraction - limited angular resolution of a telescope, in arcseconds

λ = wavelength of light, in meters

D = diameter of telescopes objective, in meters



Diffraction by a **circular aperture**. Notice the variation of intensity with angle.

Question: What is the diffraction limited angular resolution of VLBI at 1 mm assuming an effective baseline of 10,000 km?

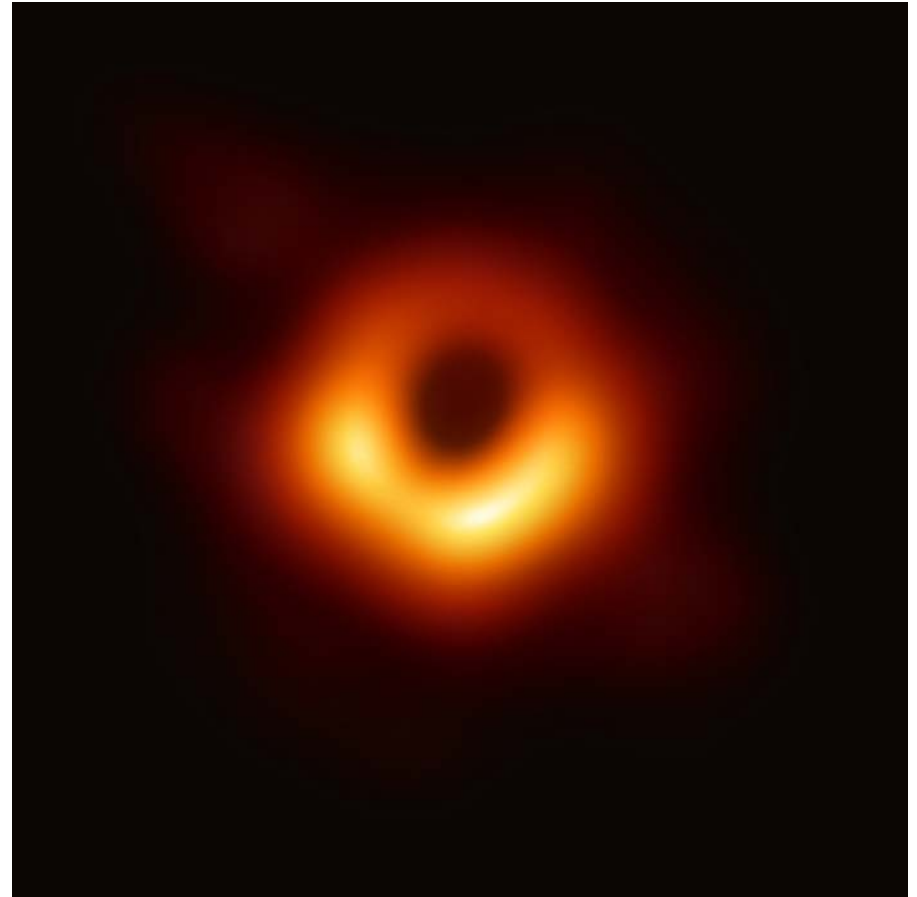
Imaging a Black Hole

The first image of a black hole, at the center of galaxy M 87, was published by the EHT Collaboration on April 10, 2019.

Observed at a wavelength of 1.3 mm and with a diffraction-limited resolution of 25 microarcseconds.

The **black hole shadow** in the middle results from light paths absorbed by the black hole.

The crescent-shaped emission ring is the black hole's **photon ring**, where gravity is so strong that photons are forced to travel in orbits.



EHT image of M87

Imaging a Black Hole

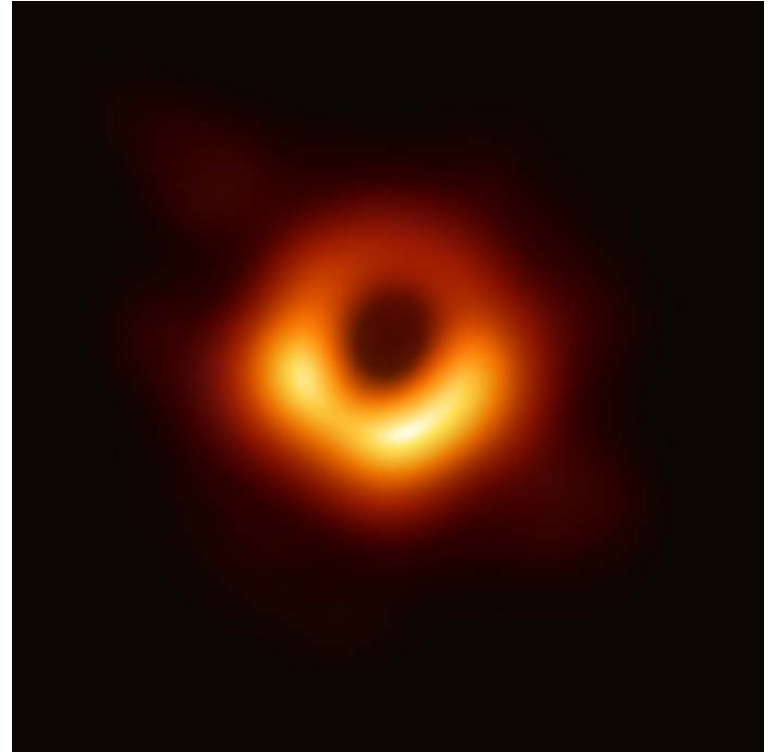
For M87, the **crescent angular diameter** is:

$$d = a\theta_g = a \frac{GM}{c^2 D} = a \frac{r_g}{D},$$

where $a \sim 9.6-10.4$, d is in radians,
and D is the distance to M87

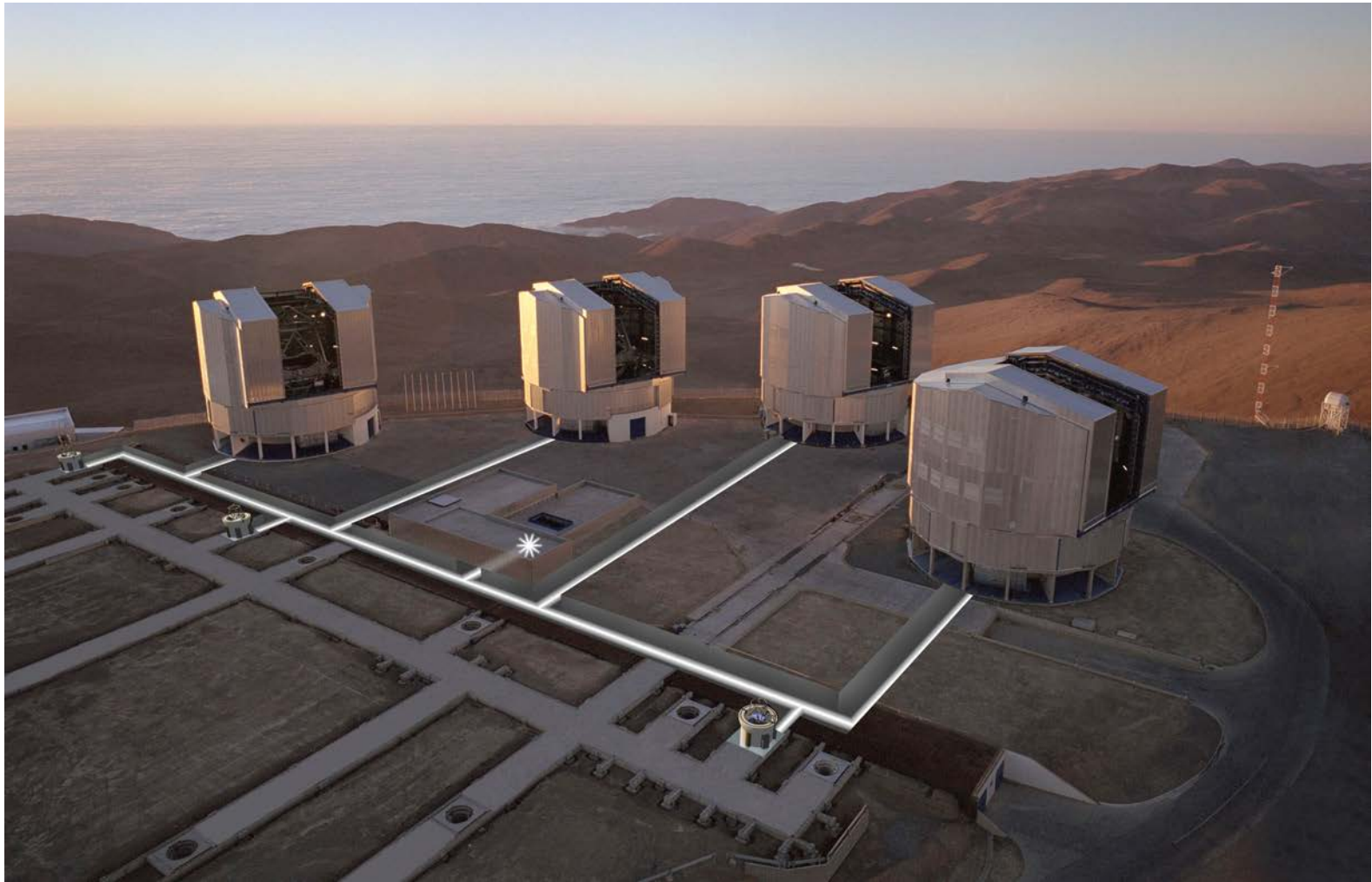
The observed ring diameter is
 $d = 42 \pm 3$ micro-arcsec (μas)
and the distance to M87 is:
 $D = 16.8 \pm 0.8$ Mpc

Solving for the mass of the black hole:
 $M = 6.5 \times 10^9 M_\odot$



EHT image of M87

Interferometry with the Very Large Telescope

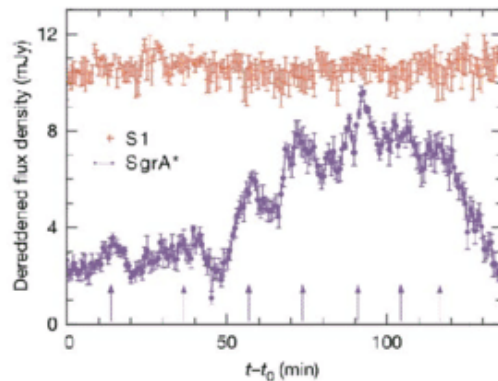


Beams from four 8.2 meter telescopes are combined to form an interferometer operating in the infrared. Imaging Resolution \sim a few milli-arcsec

Interferometry with the Very Large Telescope

The VLA Gravity instrument has been used to follow the peaks of infrared emission that appear to orbit SgrA* with a period of ~ 45 minutes.

One interpretation is that a “hot spot” is orbiting the black hole at a distance of about 2 times the black hole shadow.



Left: Light curve of the first NIR flare from Sgr A*, showing a characteristic substructure, denoted by the arrows at the x-axis. Right: Artist's impression of a flare orbiting at the last stable orbit around the SMBH.

Interferometry with the Very Large Telescope

The VLA Gravity instrument has been used to follow the peaks of infrared emission that appear to orbit SgrA* with a period of ~ 45 minutes.

One interpretation is that a “hot spot” is orbiting the black hole at a distance of about 2 times the black hole shadow.

$$\frac{mv^2}{r_{hot}} = \frac{GmM_{BH}}{r_{hot}^2} \rightarrow v^2 = \frac{GM_{BH}}{r_{hot}} \rightarrow \frac{4\pi^2 r_{hot}^2}{P^2} = \frac{GM_{BH}}{r_{hot}} \rightarrow r_{hot}^3 = \frac{GM_{BH}P^2}{4\pi^2} \rightarrow$$
$$r_{hot} = \left(\frac{GM_{BH}P^2}{4\pi^2} \right)^{1/3} \rightarrow r_{hot} \approx 7.5r_g$$

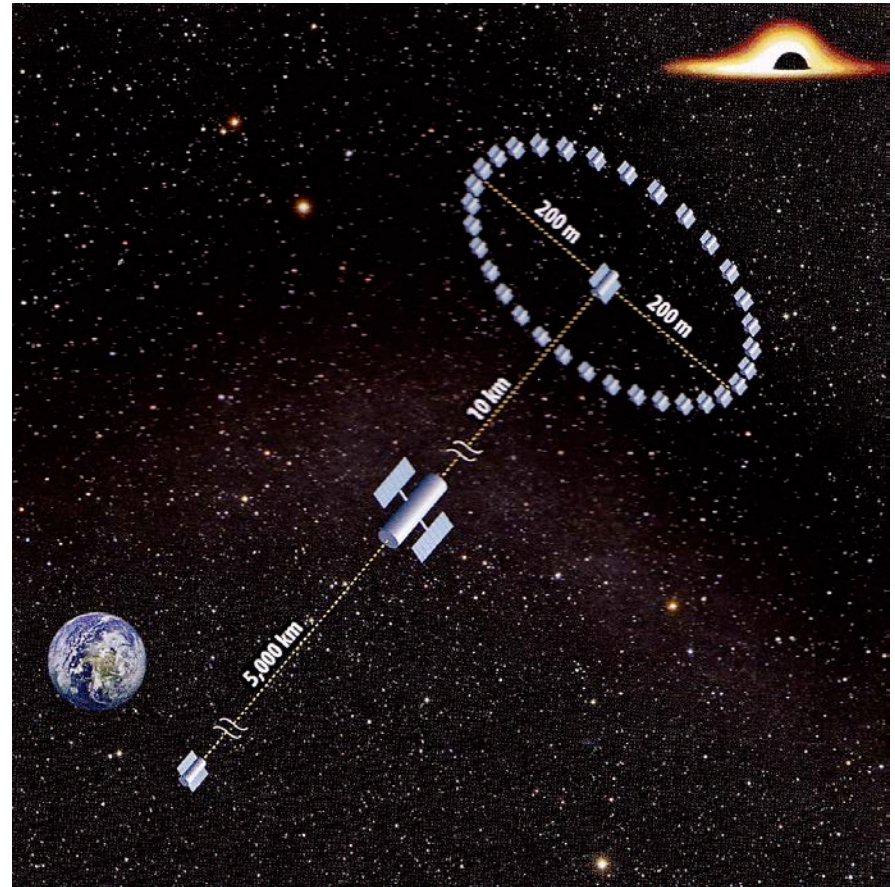
More complex analyses that include relativistic effects find that the hot spots are orbiting at $r_{hot} \sim 9 r_g$ with an inclination of $i \sim 140^\circ$.

Predicted Distortion of a BH Accretion Disk

Observations of Sgr A*

30 mirror satellites fly in tight formation around a hub spacecraft. 30 combining mirrors are in the converger spacecraft which can then direct X-rays to interfere on the detector spacecraft.

The angular resolution of this instrument will be 0.1 microarcseconds.

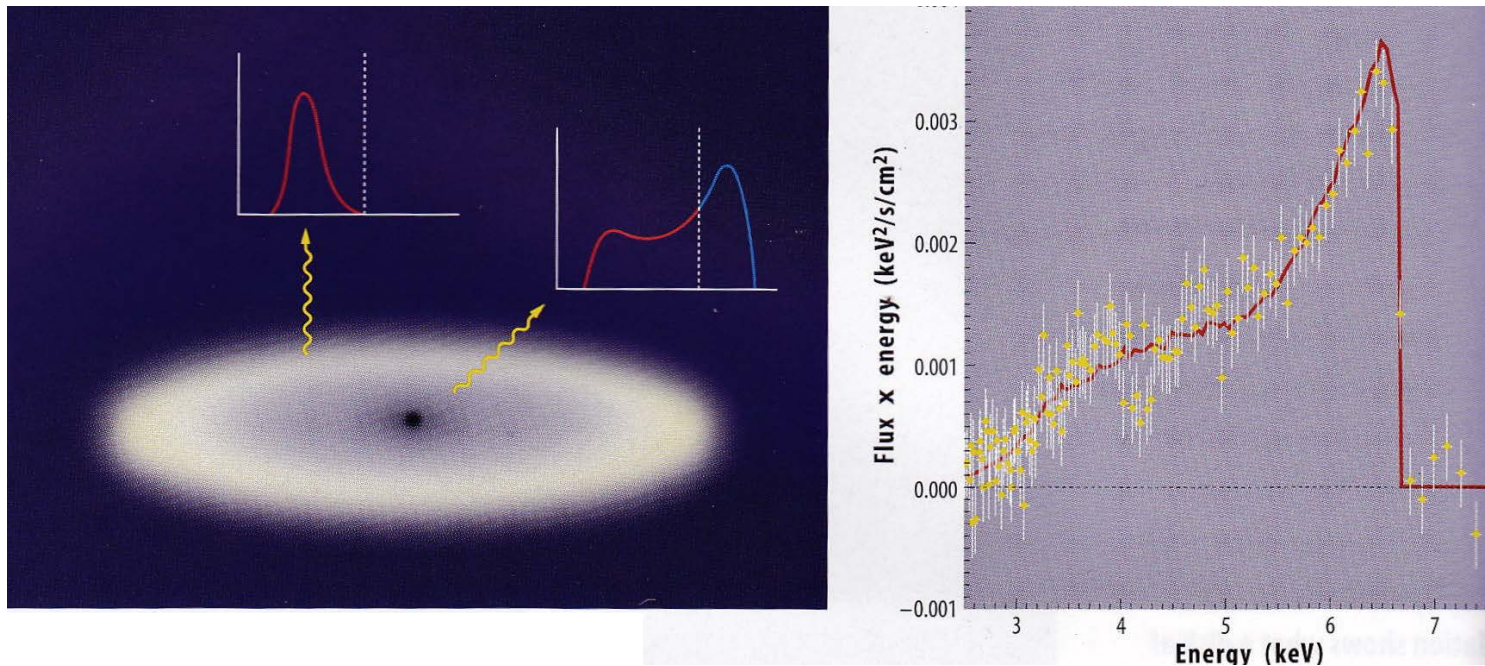


Concept: Micro-Arcsecond X-ray Imaging Mission (**MAXIM**)

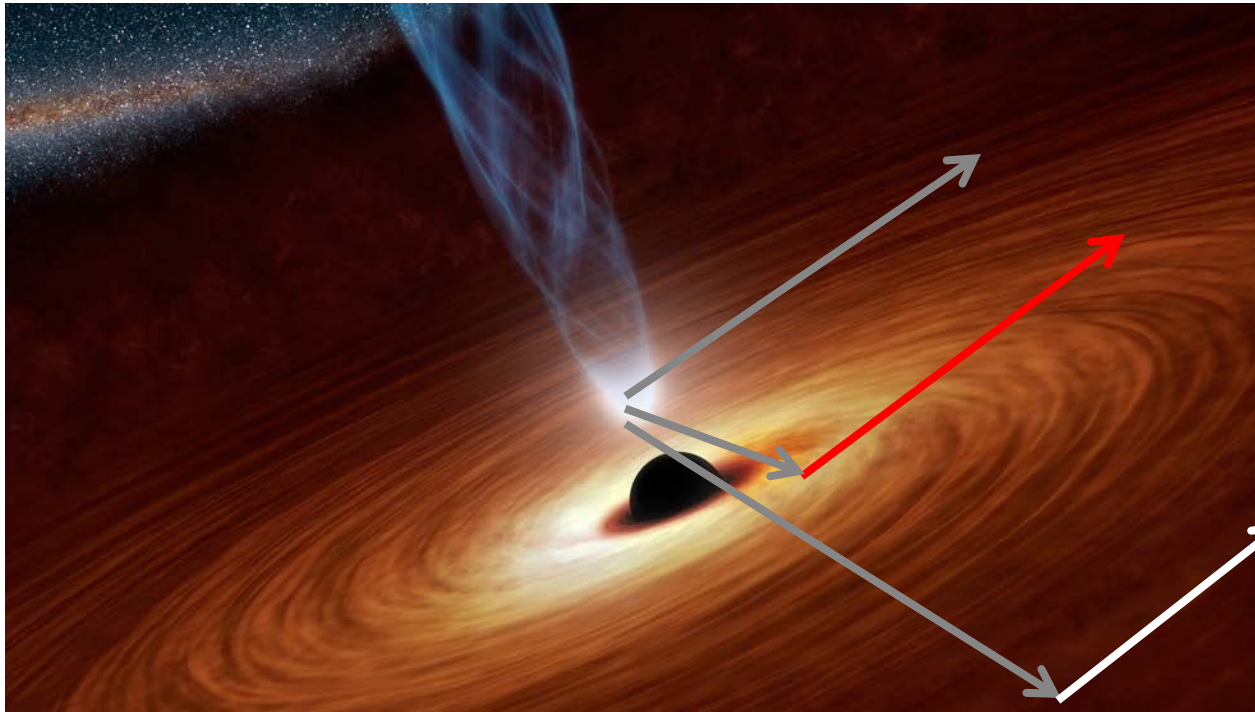
Predicted Distortion of a BH Accretion Disk

Distortion of fluorescent Fe Line

The spectrum of the accretion disk originating close to the black hole will be distorted as predicted by special and general relativity. Fits to the observed Fe line profile with Fe line models provide constraints of the inner and outer radii of the emitting region, the inclination angle and the spin of the black hole.



Fiducial Model



**X-ray Power-Law
from compact
corona**

**Relativistically
Blurred Reflection
(line + continuum)**

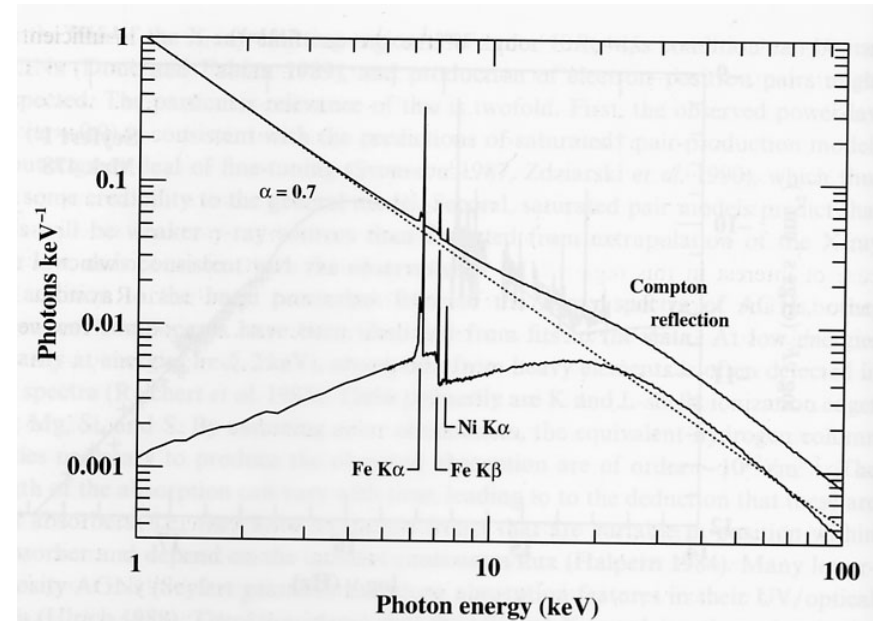
**Distant Reflection
(line + continuum)**

Geometrically thin, optically thick accretion
disk emitting primarily in UV/Optical

Predicted Distortion of a BH Accretion Disk

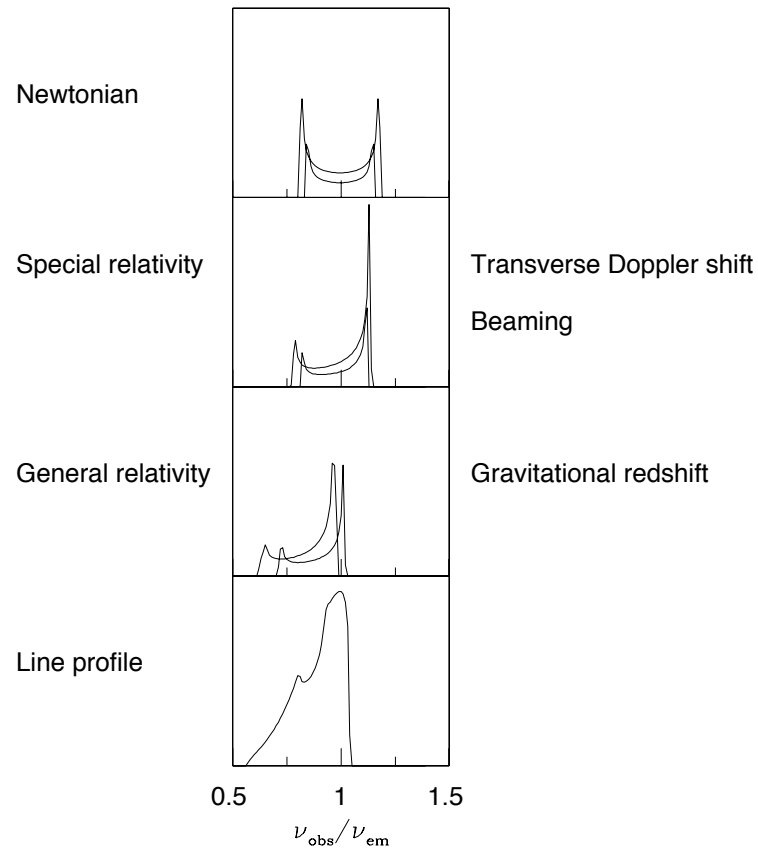
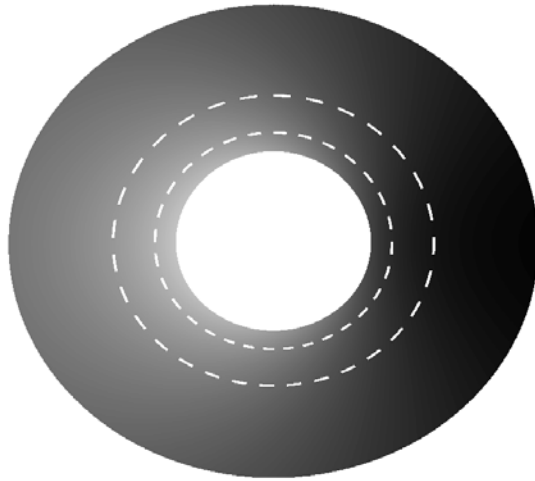
The fluorescence Fe line is produced by the scattering of X-rays from a cold or ionized accretion disk. The source of the hard X-rays is thought to be a hot corona near the black hole.

The energy of the Fe fluorescence line depends on the ionization level of the Fe. For near neutral Fe the line is at 6.4keV.



An X-ray reflection model. An input power law continuum with energy index $a=0.7$ (dotted line) irradiate a cold slab of gas. The lower spectrum shows the reflected X-ray spectrum. The combination of the Compton down-scattering (high-energy photons lose energy as they recoil off the electrons in the disk) and the photoelectric absorption of lower-energy photons in the disk results in a hump between ~ 20 -100 keV. Courtesy of Ian George.

Predicted Distortion of a BH Accretion Disk



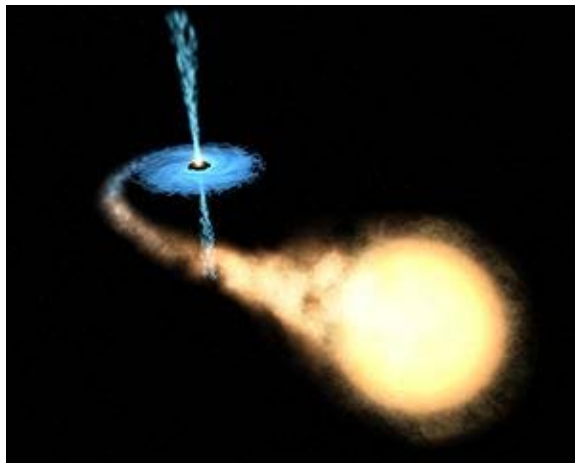
Line broadening from an intrinsically narrow line emitted from two radii in an accretion disk. The lowest panel shows the result obtained by summing many disc radii, weighted by the expected emissivity. Courtesy of A. C. Fabian, [astro-ph/0103438](https://arxiv.org/abs/astro-ph/0103438)

The line is distorted due to Doppler broadening, special relativistic effects of beaming, the transverse Doppler effect and the general relativistic effect.

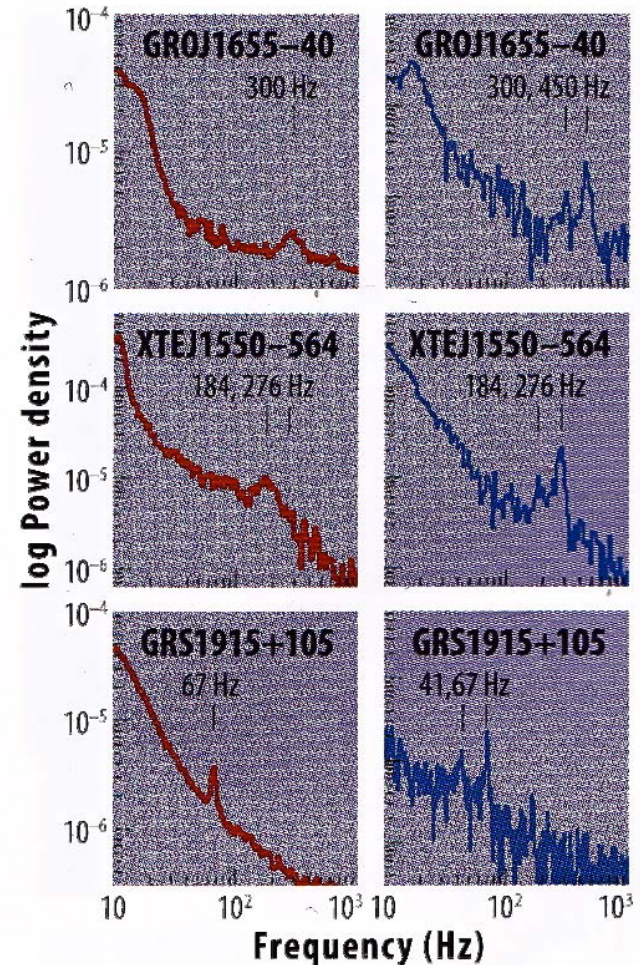
Testing Gravity with Quasi-Periodic Oscillations

Nearly periodic signals called quasi-periodic oscillations (**QPOs**) have been detected in the X-ray light – curves of X-ray binaries.

An X-ray binary is thought to be comprised of a normal star (donor star) and a compact object (black hole, neutral star or white dwarf). The X-rays in an X-ray binary come from the compact object.



Artist's impression of an X-ray Binary



The QPO frequencies remain unchanged as the spectrum and intensity varies between the left and right panels.

Testing Gravity with Quasi-Periodic Oscillations

Scientists are still trying to figure out whether QPOs can tell us something about the geometry of space near a black hole or neutron star.

Possibilities:

- The strong curvature near the center of the BH traps and amplifies **oscillations of the accretion disk.**
- The inner part of the accretion disk may be precessing
- Hot spots on the accretion disk

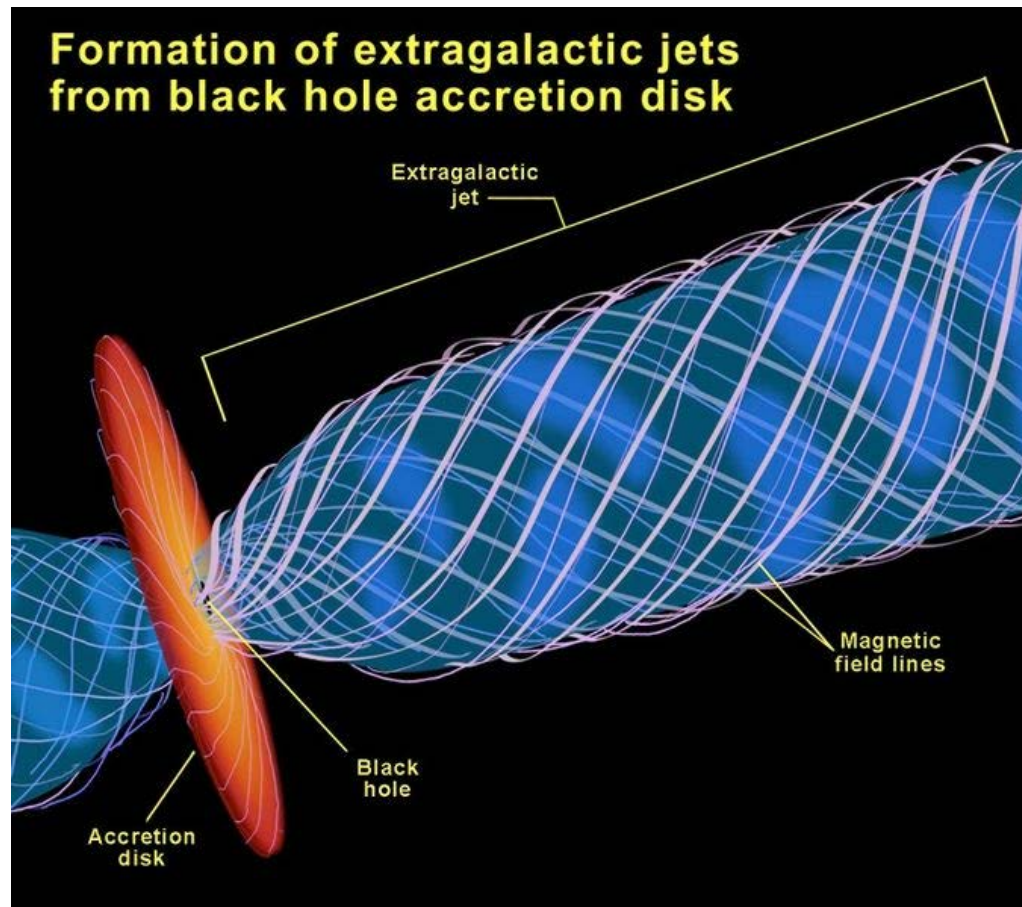
Testing Gravity with Jets

Roger Penrose first suggested that the **energy of spinning black holes can be tapped** to power fast outflowing jets from the centers of AGNs. One test of this theory is to see if the properties of jets correlate with the spin of their jets.



Testing Gravity with Jets

The presence of spinning black holes is supported by the steady orientation of jets for many hundreds of millions of years. It is thought that the spinning black hole acts as a gyroscope to enforce the stability of the jet.



Testing Gravity Near a Spinning BH

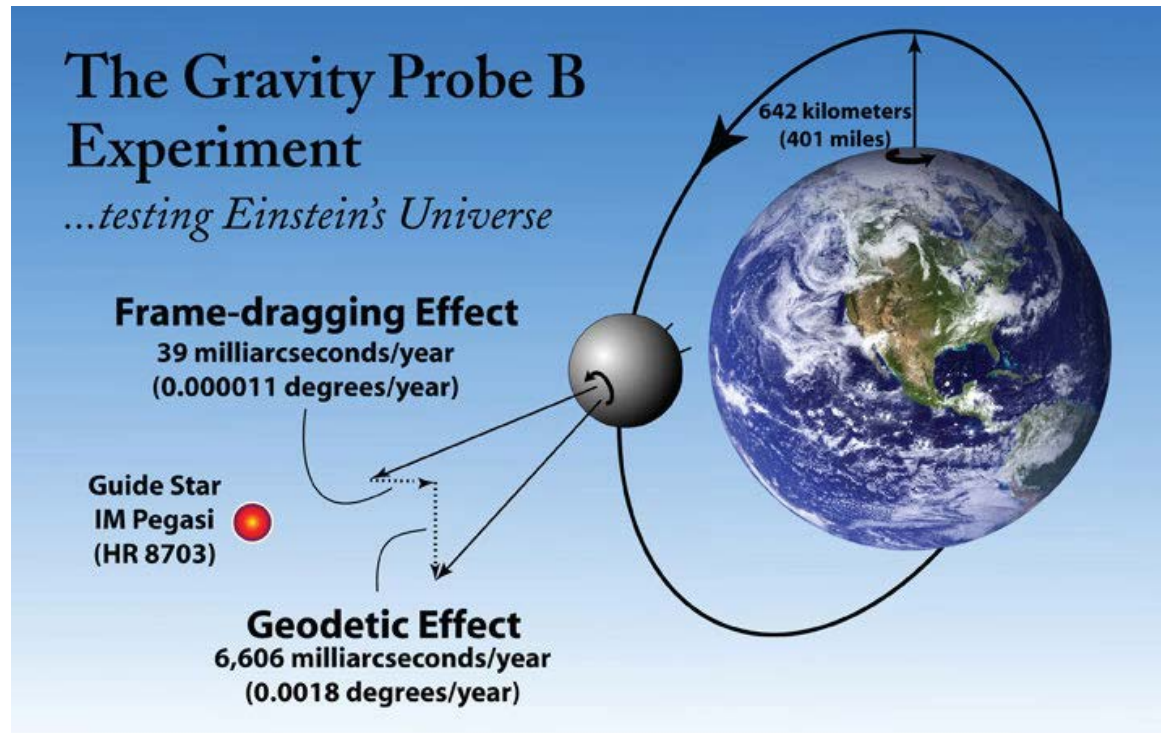
Lense and Thirring using Einstein's equations first **predicted that the rotation of objects drag space and time around themselves** (frame-dragging).

A gyroscope placed near a spinning BH will maintain its orientation with respect to an “inertial frame” in its vicinity, but this inertial frame will rotate with respect to a distant observer.

This means that a distant observer will see the gyroscope's axis slowly change orientation with time.

Space itself is dragged in the same direction as the BH and the closer to it the faster it is dragged.

Experimental Tests of Frame-Dragging

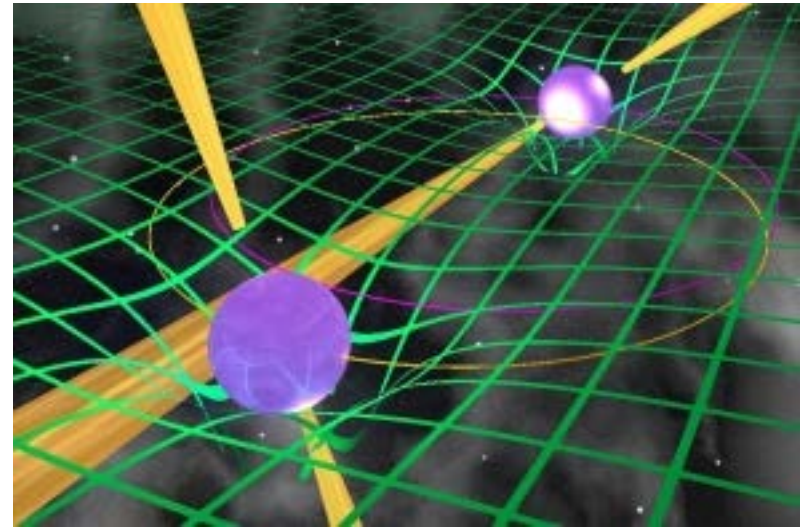


Gravity Probe B spacecraft was launched in April 2004 in a polar orbit.

Gravity Probe B carried four gyroscopes and one telescope, and was designed to measure the relativistic precessions of the four test-gyroscopes with respect to the distant star IM Pegasi.

Experimental Tests of Frame-Dragging

The geodetic precession of the spin axis of a binary pulsar, a spin-orbit frame-dragging effect, has been observed in the binary system PSRB1534+112, where its measured value has been reported to be $\sim 0.44^\circ$ (+0.48,-0.16)/year in agreement with the GR prediction of 0.51° /year.



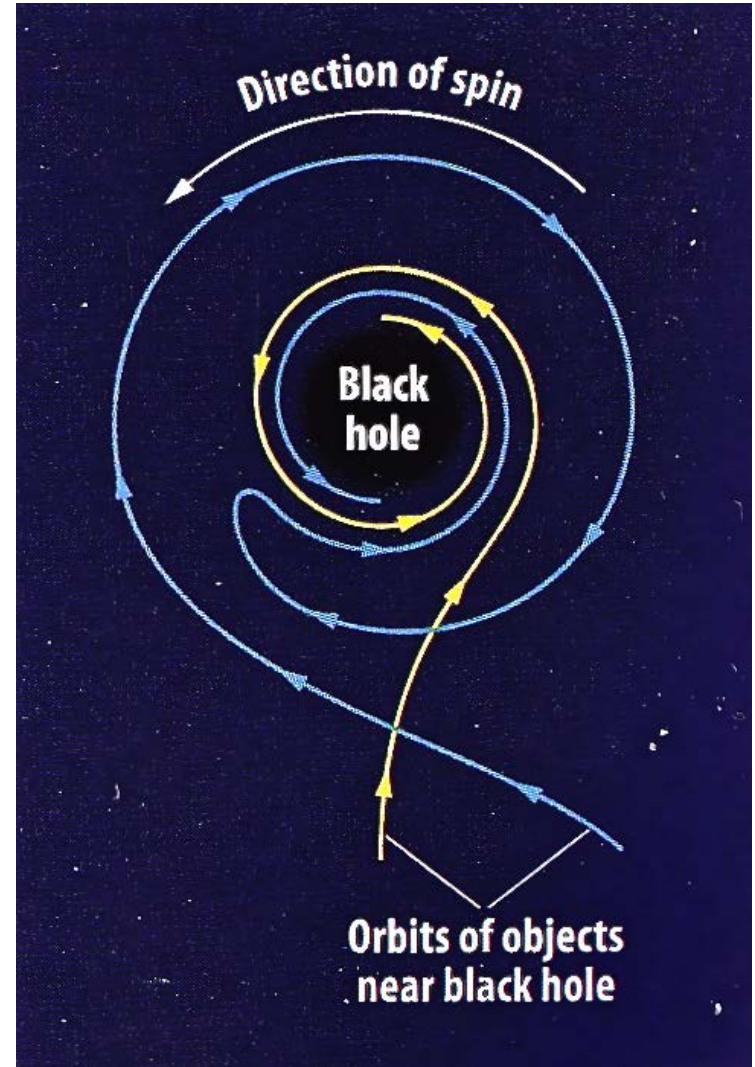
Since 1974, a number of binary pulsars have been discovered and they provide extraordinary astrophysical laboratories for testing the general theory of relativity via the measurement of their orbital parameters.

Testing Gravity Near a Spinning BH

The path of an **object falling toward a Kerr black hole** is influenced by the rotation of the hole, shown here as counterclockwise.

The **yellow particle approaches along a radial trajectory**, but its path is deflected into a counterclockwise spiral.

The **blue particle approaches on a clockwise orbit**; a distant observer sees its direction reverse so that it too spirals into the hole on a counterclockwise direction.



Spinning Black Hole

Two surfaces are important in describing Kerr black holes:

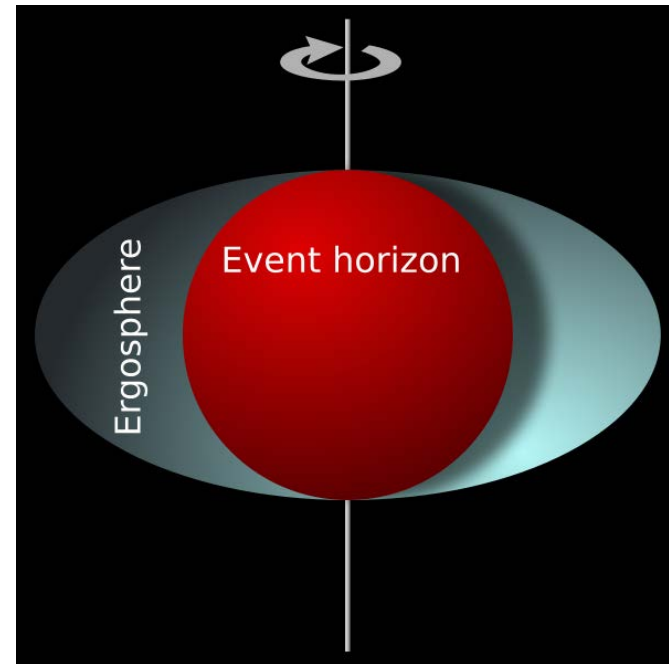
-The Static limit

Once inside the static limit even light has to rotate in the same direction as the black hole

-The Horizon

Within the inner surface light cannot escape.

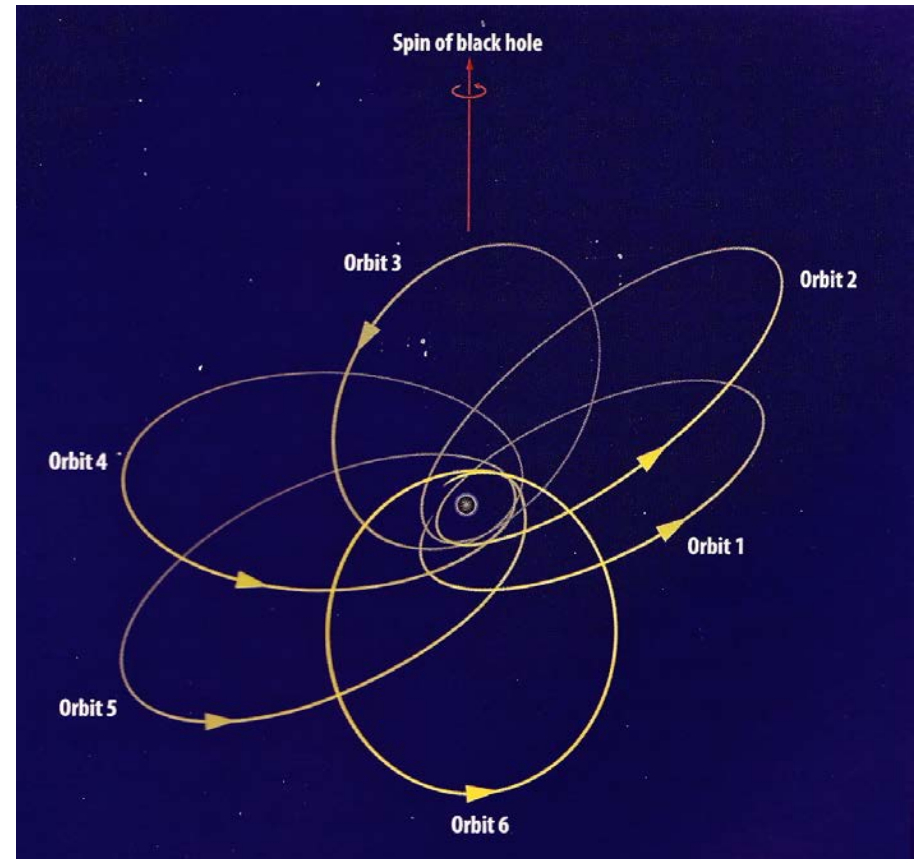
The region between the static limit and the horizon is called the **ergosphere**. Processes in this region can extract energy from the black hole.



Testing Gravity Near a BH

The orbits of stars near SMBHs should show strong deviations from classical Newtonian orbits because of the geodetic and L-T effect.

These effects might be noticeable in orbits of stars around $10^{9-10} M_{\odot}$ SMBHs where the tidal forces near the horizon are weaker compared to less massive SMBHs.

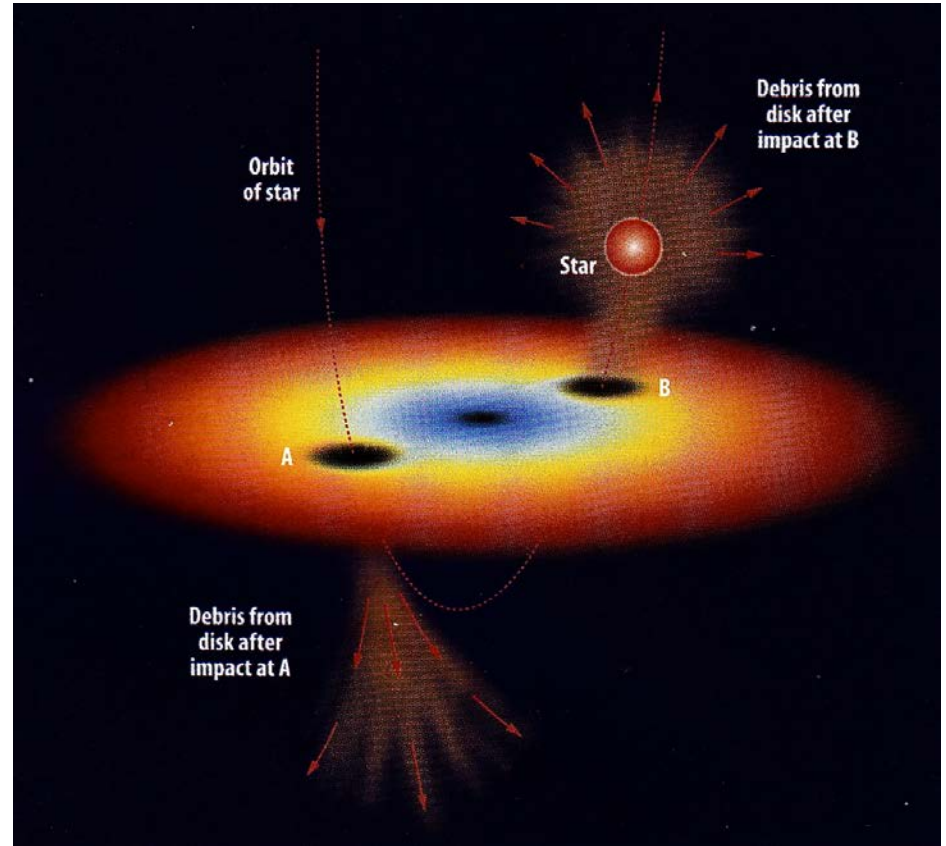


Simulated orbits of stars around a spinning SMBH. GR effects will produce complicated orbital trajectories.

Testing Gravity Near a BH

As stars crash through the accretion disk they will produce debris that can lead to periodic obscuration of the disk.

By measuring the expected periodic variability induced by the crash scientists hope to infer the properties of the orbits of stars near the SMBH and test GR.



A star crashing through twice each orbit through the accretion disk will lose angular momentum and will gradually drift closer to the SMBH.

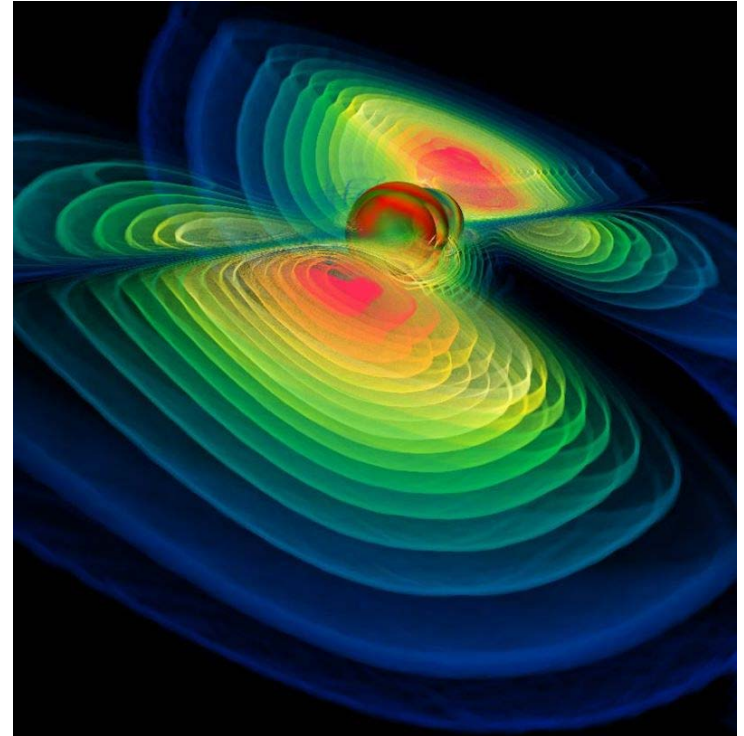
Gravitational Waves

A star orbiting a black hole loses energy through gravitational radiation and will eventually spiral into it even if there are no other energy losses.

But what is gravitational radiation?

Changes in the shape and size of an object in general will produce a change in the gravity around the object.

Gravitational waves are waves of spacetime that travel at the speed of light. They are very weak compared to electromagnetic waves. The electromagnetic force between massive objects is usually very weak because electric charge comes in positive and negative flavors that cancel out. Gravity becomes important at large scales because there are no “negative” masses to cancel out gravity.

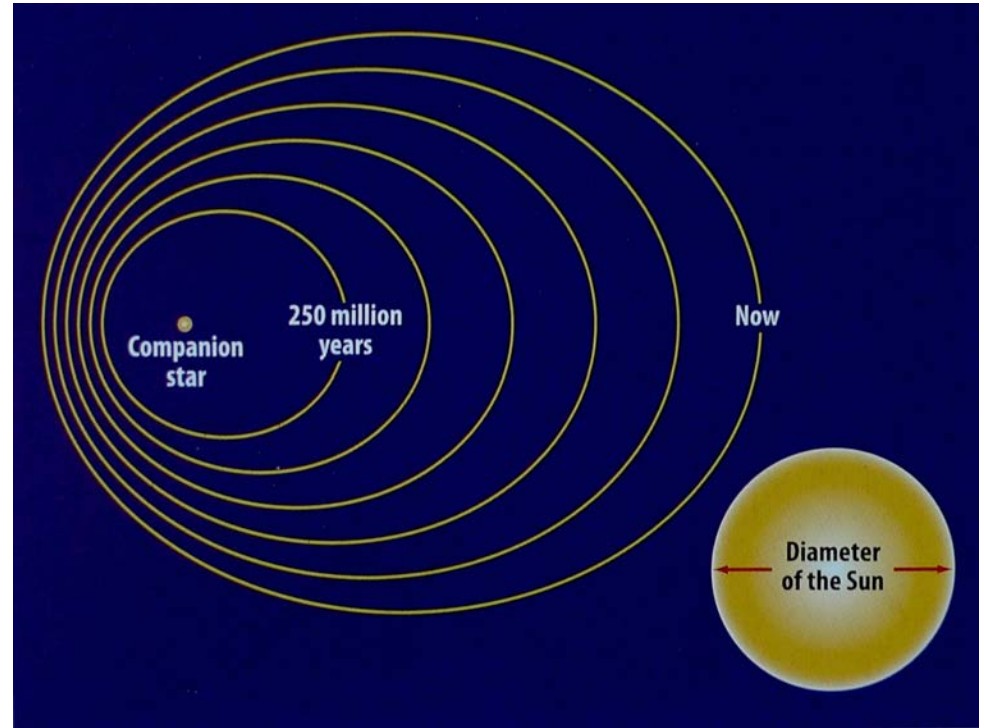


Gravitational Radiation

In 1974 Russell Hulse and Joseph Taylor measured that the orbital period of a pulsar in a binary system decreases very slowly due to a shrinkage of the orbit by a few cm per year.

The period of the orbit is now ~ 7.75 hours

The **merger of the pulsar and its companion neutron star** is predicted to take $\sim 3 \times 10^8$ years.



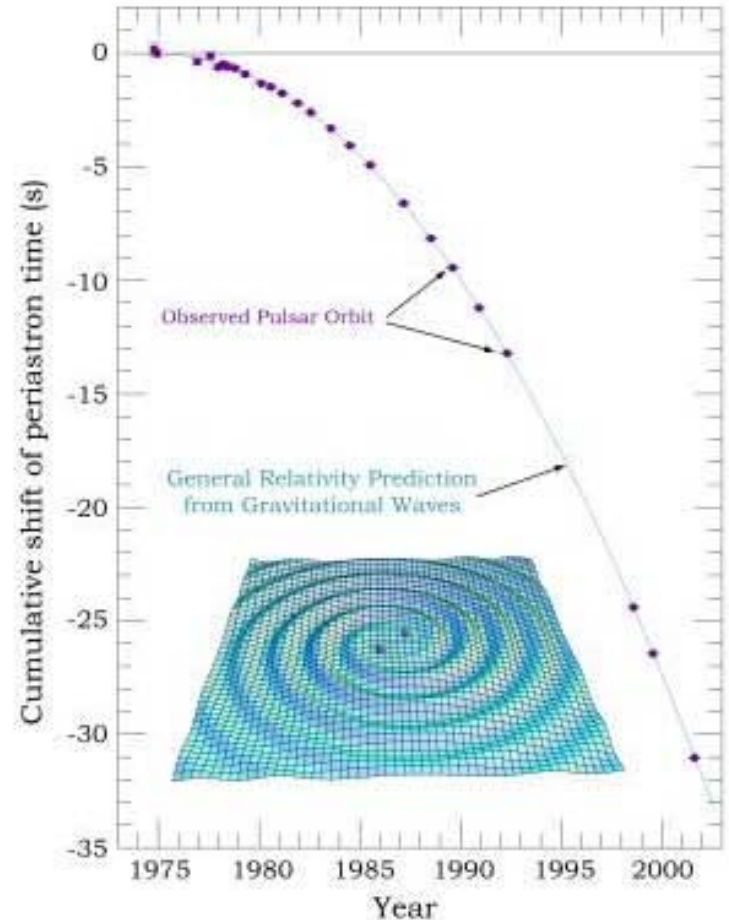
The Hulse-Taylor binary pulsar follows a highly elliptical orbit around its companion. Because of gravitational radiation the orbit becomes smaller and more circular.

Gravitational Radiation

Shrinkage of the orbit is measured by timing the closest approach of the two stars using the pulsar as a clock.

By measuring the time of periastron (closest approach) Hulse and Taylor found that the shrinkage is exactly what is predicted by Einstein's theory assuming that the shrinkage is caused by energy lost through gravitational quadrupole radiation.

The power radiated in gravitational waves is $\sim 7.35 \times 10^{24}$ watts



Gravitational Radiation from Mergers

The **largest bursts of gravitational radiation** are expected to occur during the merging of neutron stars or black holes.

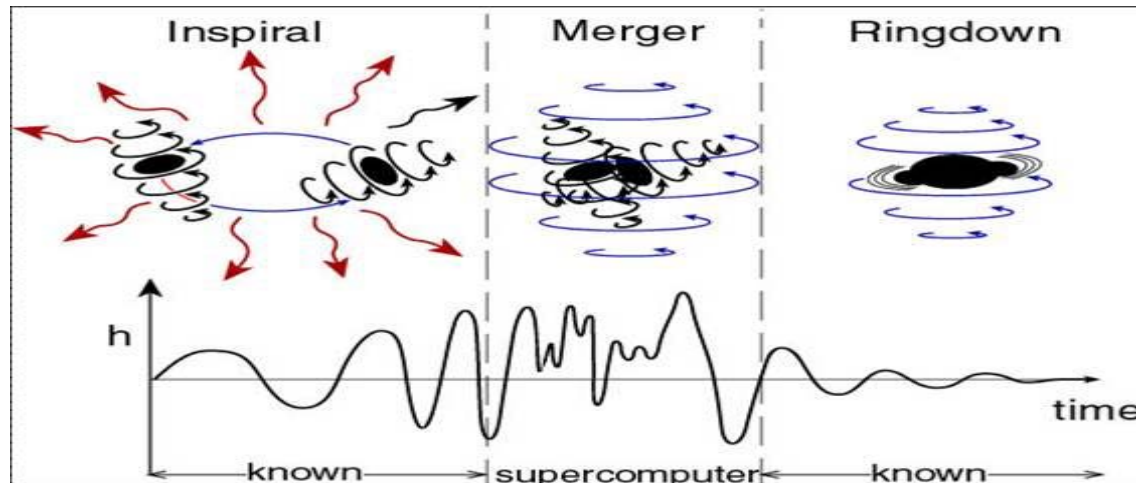
What happens when two black holes merge?

-Their separate event horizons will merge and one black hole will form.

-Stephen Hawking proved that the sum of the areas of the horizons cannot decrease.

-After the merged black hole has settled down it will become a standard black hole characterized simply by its angular momentum and mass.

Gravitational Radiation from Mergers



During the merger of two black holes the gravity field fluctuates the most. With the gravitational radiation the merging structure shakes off its hair, any asymmetries and inhomogeneities that depend on how the merger took place.

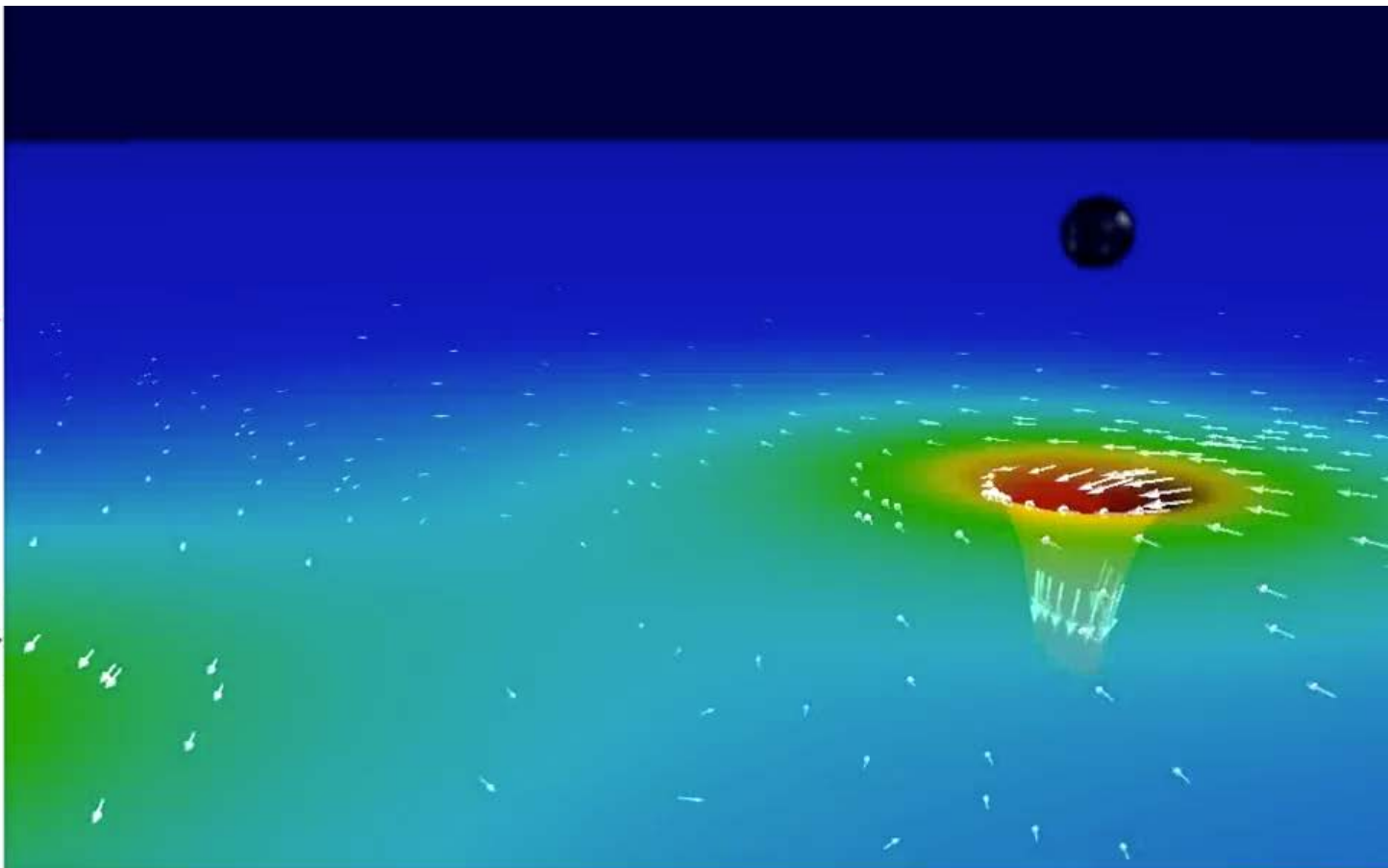
<https://www.youtube.com/watch?v=p647WrQd684>

The gravitational waves produced depend on the magnitude and relative orientations of the spins of the black holes, the impact parameters of the collision and their relative masses of the black holes.

Binary Black Hole Evolution:

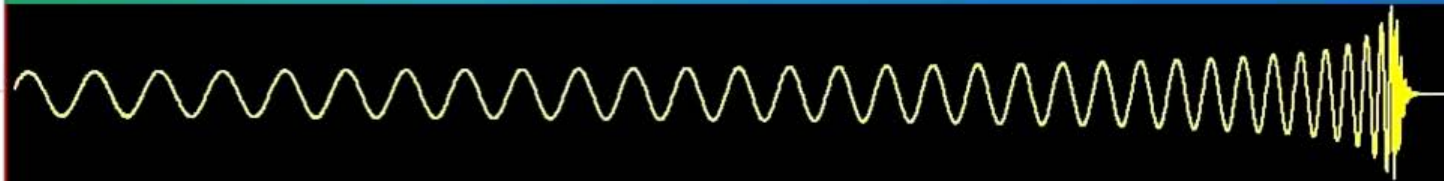
Caltech/Cornell Computer Simulation

Top: 3D view of Black Holes and Orbital Trajectory

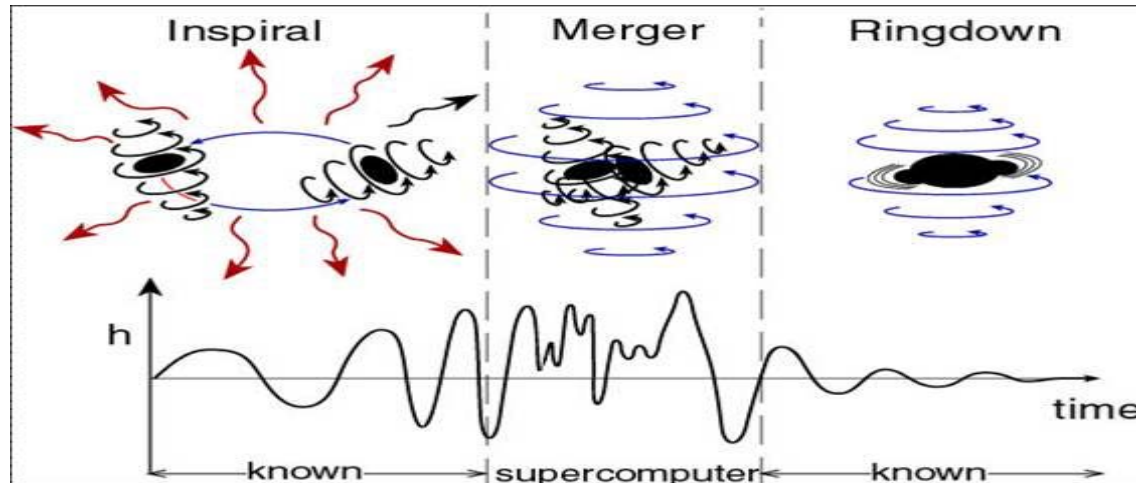


Middle: Spacetime curvature:
Depth: Curvature of space
Colors: Rate of flow of time
Arrows: Velocity of flow of space

Bottom: Waveform
(red line shows current time)



Maximum Energy Carried Away by Gravitational Radiation of Merger

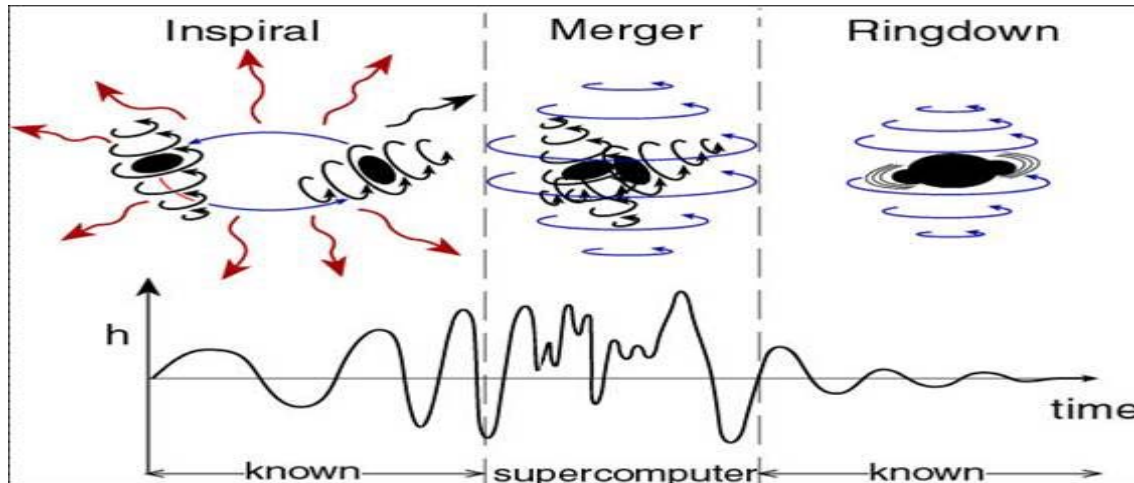


Hawking showed that the sum of the areas of the horizons cannot decrease: $A_{combined} \geq A_1 + A_2$

This implies that the maximum mass that can be carried away during the merger of two non-spinning black holes of equal mass M_{BH} is :

$$M_{max} = (2 - \sqrt{2})M_{BH}$$

Maximum Energy Carried Away by Gravitational Radiation of Merger



$$A_{\text{Combined}} \geq A_1 + A_2 \Rightarrow 4\pi R_{\text{Combined}}^2 \geq 4\pi R_1^2 + 4\pi R_2^2 \Rightarrow$$

$$\left(\frac{GM_{\text{Combined}}}{c^2}\right)^2 \geq 2\left(\frac{GM_{\text{BH}}}{c^2}\right)^2 \Rightarrow M_{\text{Combined}} \geq \sqrt{2}M_{\text{BH}}$$

$$M_{\text{radiated}} = 2M_{\text{BH}} - M_{\text{Combined}} \Rightarrow M_{\text{max,radiated}} = 2M_{\text{BH}} - \sqrt{2}M_{\text{BH}}$$

$$M_{\text{max,radiated}} = (2 - \sqrt{2})M_{\text{BH}}$$

A simulation of gravitational waves produced by two merging black holes. Colors denote the amplitude of the waves with yellow being the highest amplitude.

Simulation performed at NASA GSFC.



Detecting Gravitational Waves

A gravitational wave causes a distortion of space that travels at the speed of light. Any object in the path of the wave would feel a **tidal gravitational force that acts perpendicular to the waves direction of propagation.**

A wave intercepting a circular ring face on would distort the ring into an ellipse.

By measuring the distortion of the ring one can infer the strength of the gravitational wave.



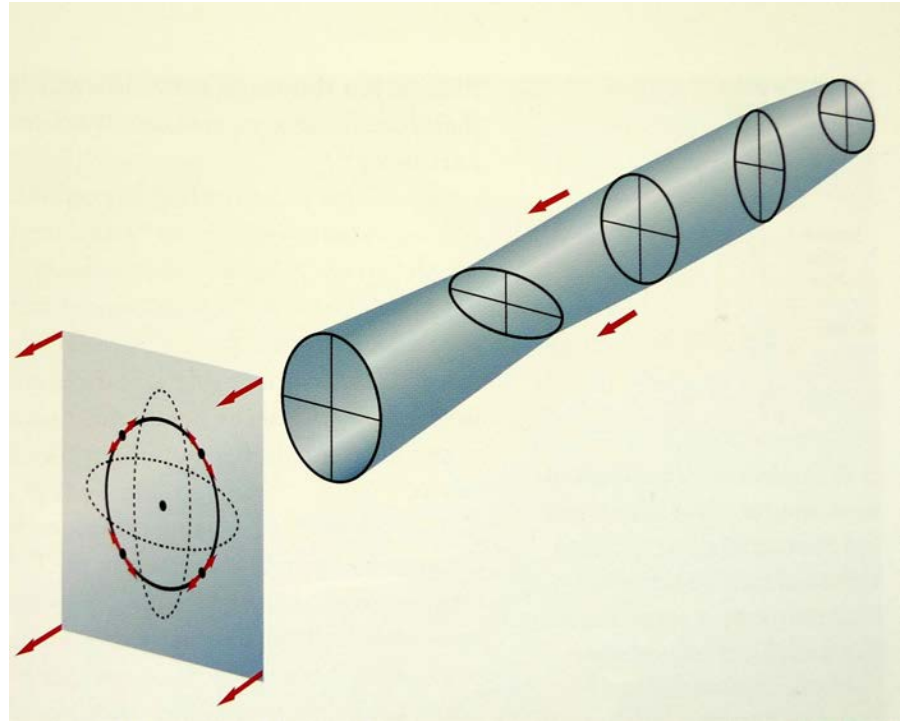
Kip Thorne

Detecting Gravitational Waves

A gravitational wave causes distortion of spacetime that travels at the speed of light.

A circular ring placed perpendicular to the direction of the wave will feel a gravitational force that is perpendicular to the direction of the waves propagation.

The waves can be detected by measuring the change of the relative positions of masses distributed around the ring.

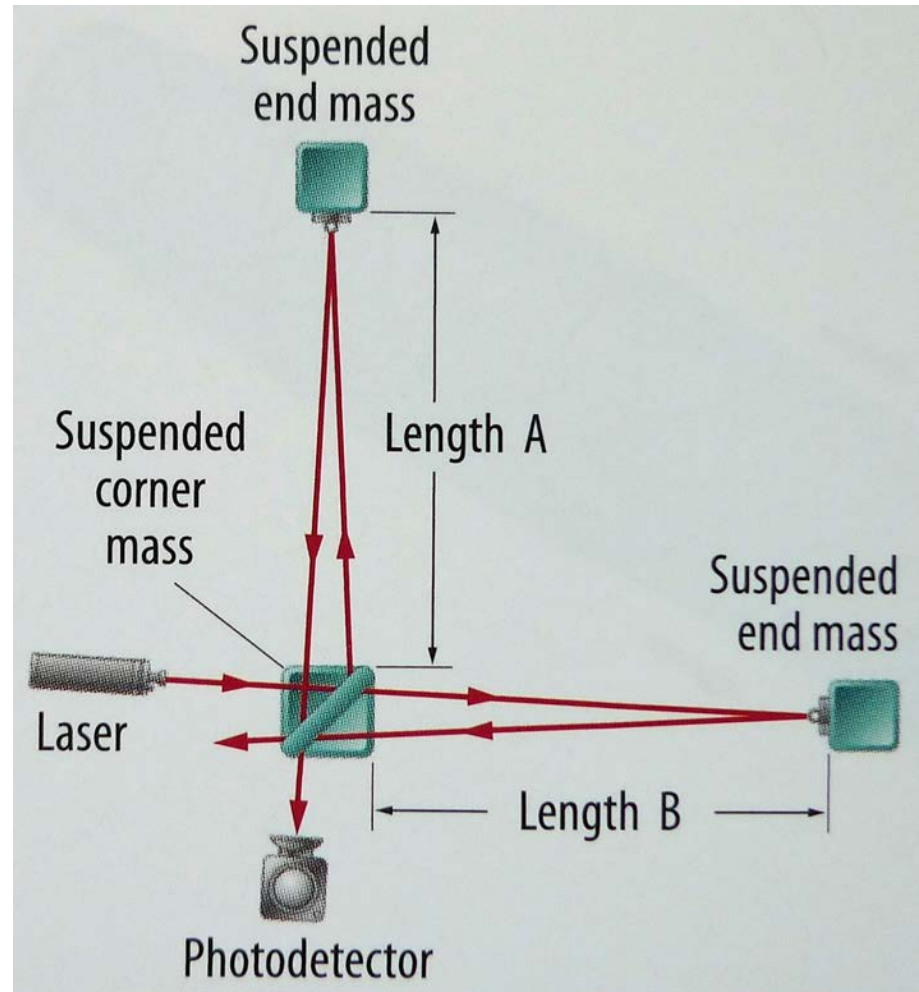


Detecting Gravitational Waves

In an interferometric gravitational wave detector a laser beam is split and reflected off mirrors attached to two end masses lying several km away.

A gravitational wave going by will in general change the lengths A and B differently.

These changes in path lengths will cause a change in the interference pattern obtained from the combined reflected ways.



Detecting Gravitational Waves



Laser Interferometer Gravitational-Wave Observatory (LIGO) near Hanford, in Washington State. A similar facility is in Livingstone, Louisiana. The goal is to detect gravitational waves from the merging of neutron star binaries, mergers of stellar mass black holes, and from supernova explosions that are sufficiently non-symmetrical.

Detecting Gravitational Waves

The **fractional change in the size (strain h)** of an object due to the gravitational waves produced when the two neutron stars in the Hulse-Taylor binary collide is $h \sim 10^{-17}$

This collision will change the distance between two particles separated by 100 m by $10^{-17} \times 100\text{m} = 10^{-15}$ m (close to the radius of a proton)

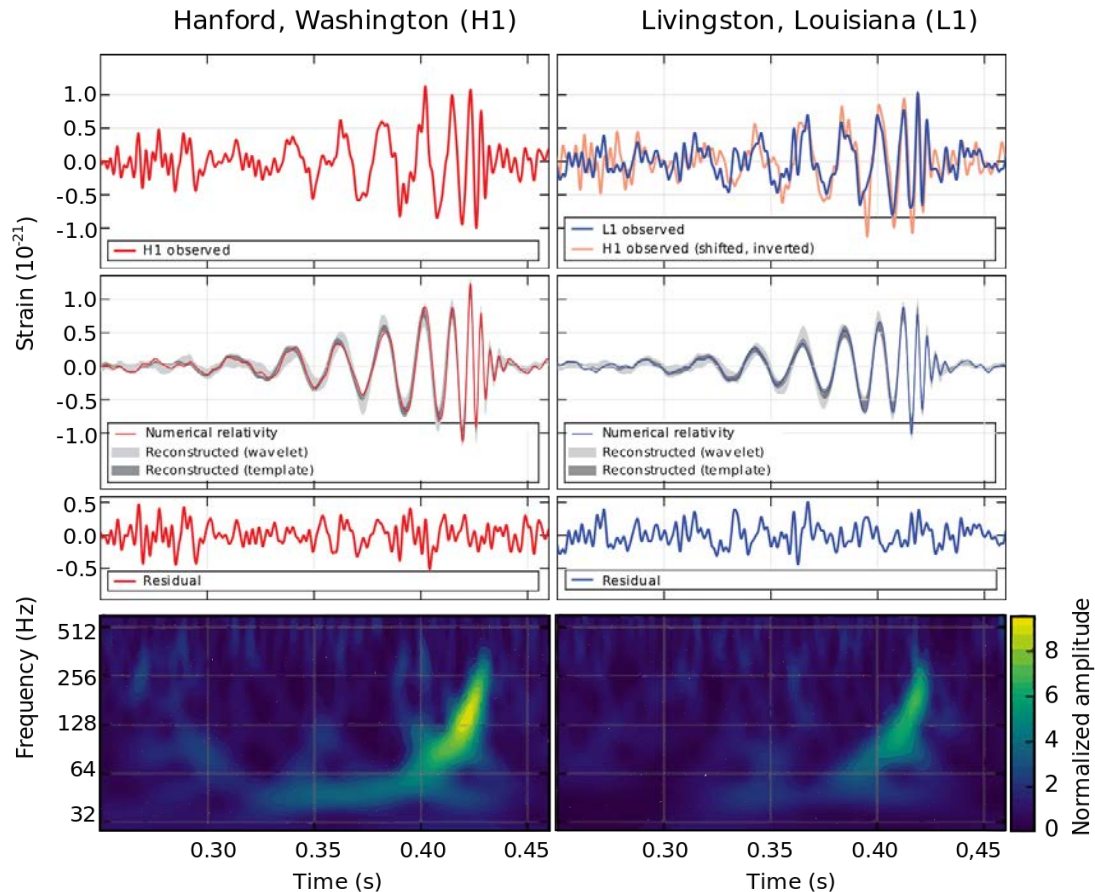
A NS-NS merger happens ~ 1 per 1×10^5 years per galaxy

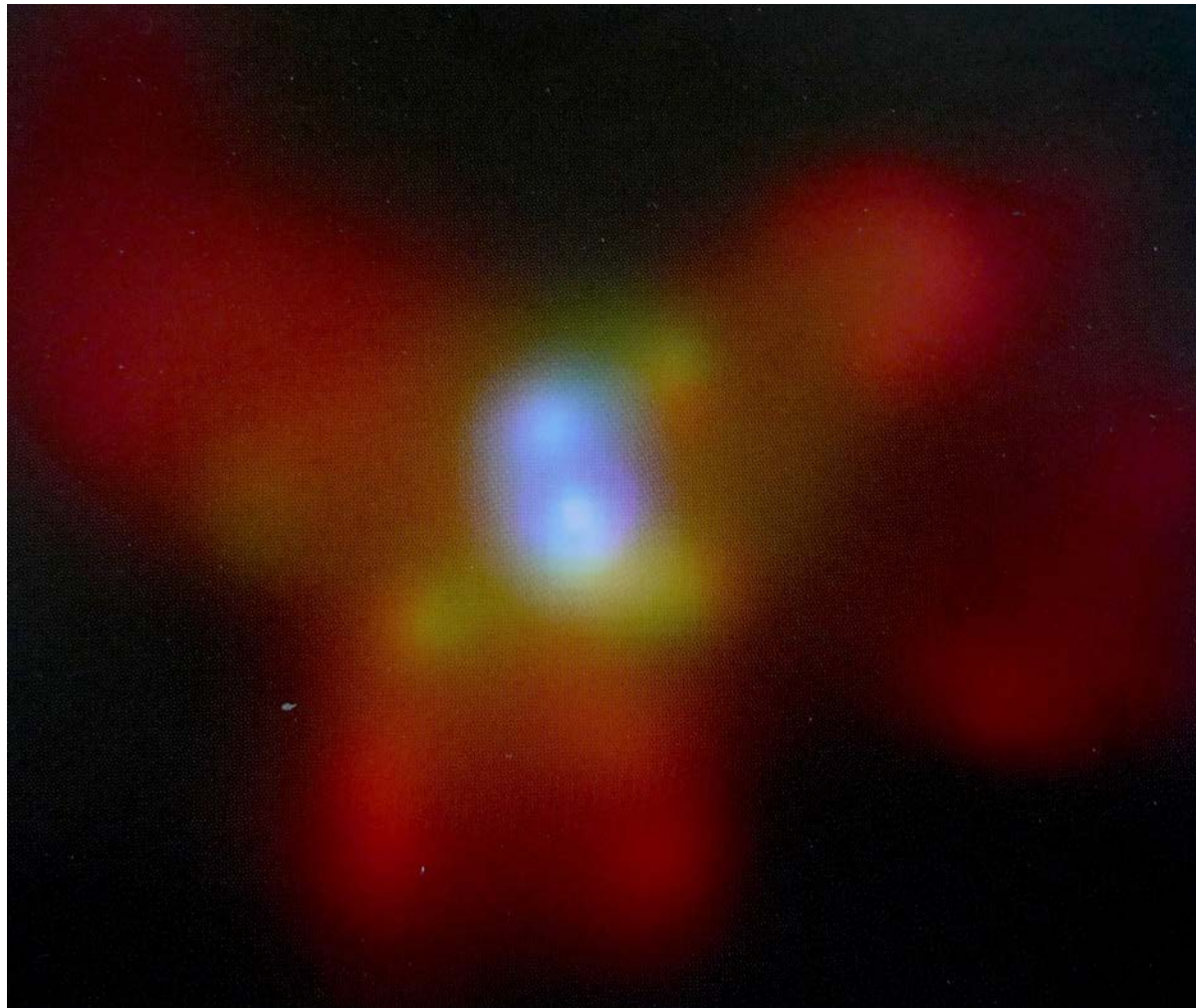
A gravitational wave detector that is sensitive enough to detect NS-NS mergers in $\sim 100,000$ galaxies will have a detection rate of **1 NS-NS merger per year.**

The **Advanced LIGO** is expected to be able to detect the merger of neutron star binaries as far as 300 million light years away. Advance LIGO will be able to measure fractional changes of size of the *arms* of the order of 10^{-21}

Discovery of Gravitational Waves

The **first direct observation of gravitational waves** was made on 14 September 2015 and was announced by the LIGO and Virgo collaborations on 11 February 2016. The gravitational wave emanated from the inward spiral and merger of a pair of black holes of around 36 and 29 solar masses (distance ~ 410 Mpc).





The galaxy NGC 6240 appears to contain two supermassive black holes that are in the process of merging.

LISA

Laser Interferometer Space Antenna



The **Laser Interferometer Space Antenna** (LISA) is a proposed mission designed to detect gravitational waves.

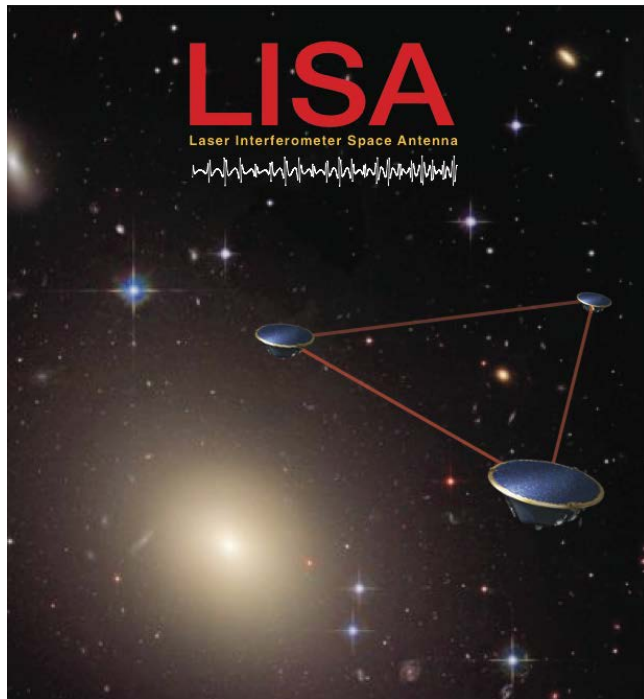
Tentative launch 2034.

LISA will have a constellation of three spacecraft, arranged in an equilateral triangle with 1 million km arms.

The distance between the satellites is monitored to detect a passing gravitational wave.

A potential source for signals are merging massive black holes at the centre of galaxies and extreme mass ratio inspirals.

LISA should be able to measure relative displacements with a resolution of 20 picometers (10^{-12} m)



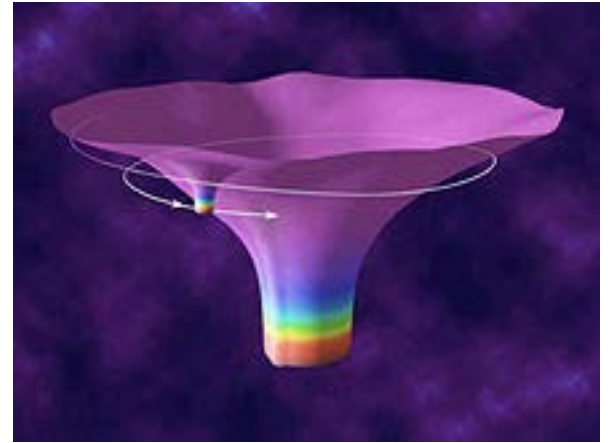
An **extreme mass ratio inspiral** (EMRI) is the orbit of a light object around a much heavier (by a factor 10,000 or more) object, that gradually decays due the emission of gravitational waves.

EMRIs are likely to be found in the centers of galaxies, where stellar black holes and neutron stars, may be found orbiting a supermassive black hole.

EMRIs are one of the most promising sources for gravitational wave astronomy using LISA.

If such signals are successfully detected, they will allow accurate measurements of the **mass and angular momentum of the central object**, which in turn gives crucial input for models for the formation and evolution of supermassive black holes.

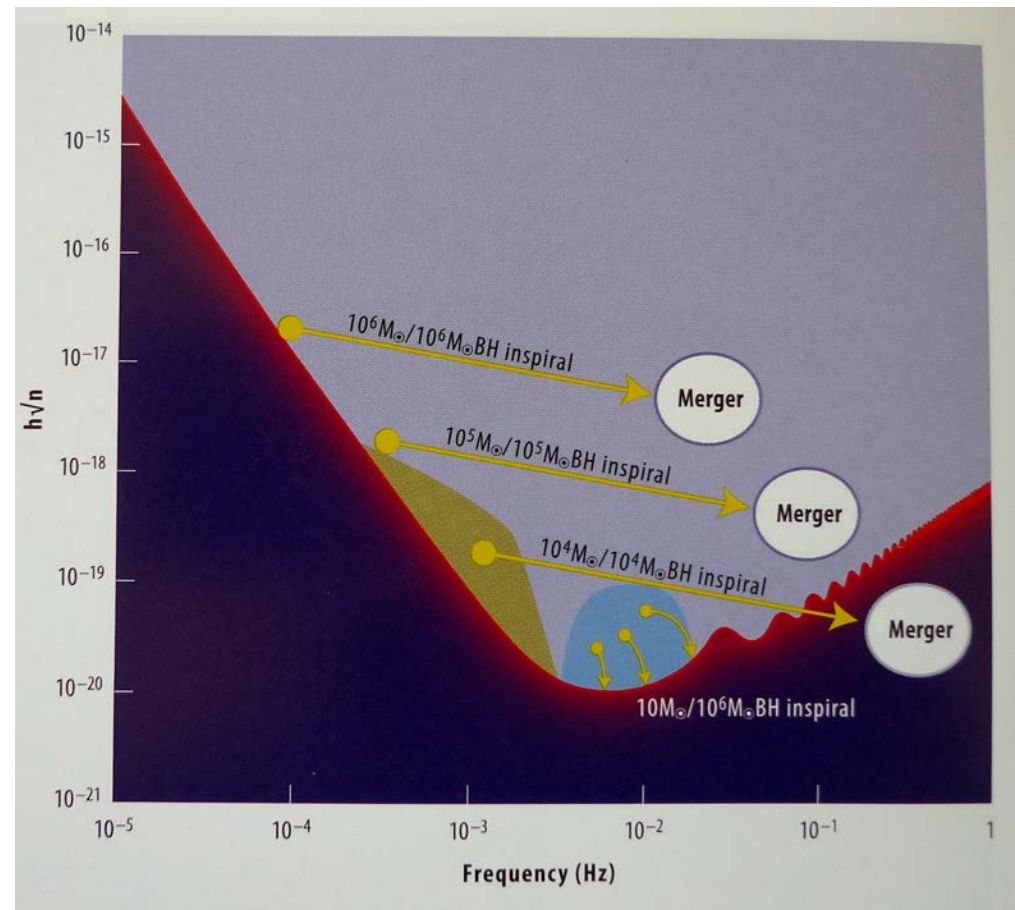
Moreover, the gravitational wave signal provides a **detailed map of the spacetime geometry** surrounding the central object, allowing unprecedented tests of the predictions of general relativity in the strong gravity regime.

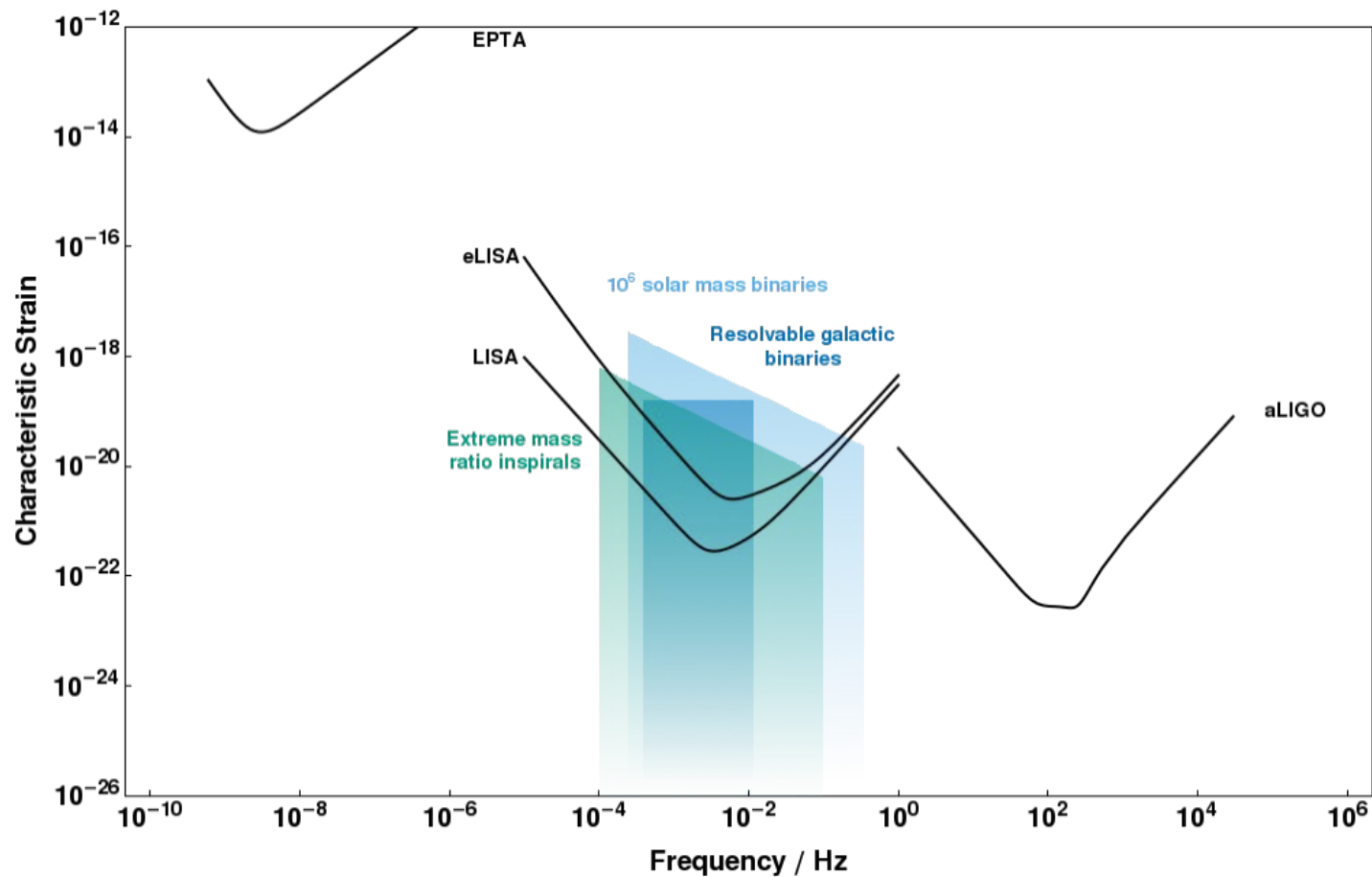


Artist impression of the spacetime generated by an extreme mass ratio inspiral.

Predicted sensitivity of LISA to various gravitational wave sources.

The orange bands shows the minimum signals that should be detectable as a function of frequency.





Detector noise curves for LISA and eLISA as a function of frequency. They lie in between the bands for ground-based detectors like advanced LIGO (aLIGO) and pulsar timing arrays such as the European Pulsar Timing Array. The characteristic strain of potential astrophysical sources are also shown. To be detectable the characteristic strain of a signal must be above the noise curve.

<https://www.youtube.com/watch?v=uznU6UVPong>

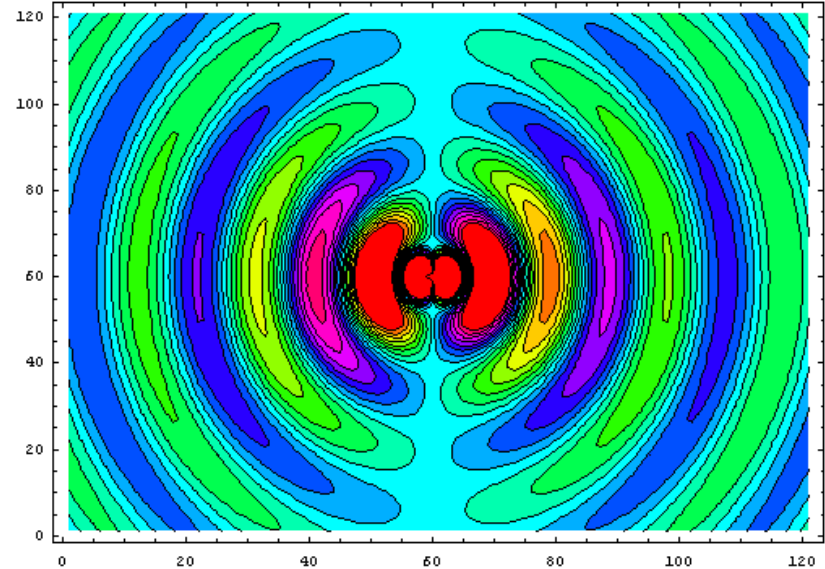
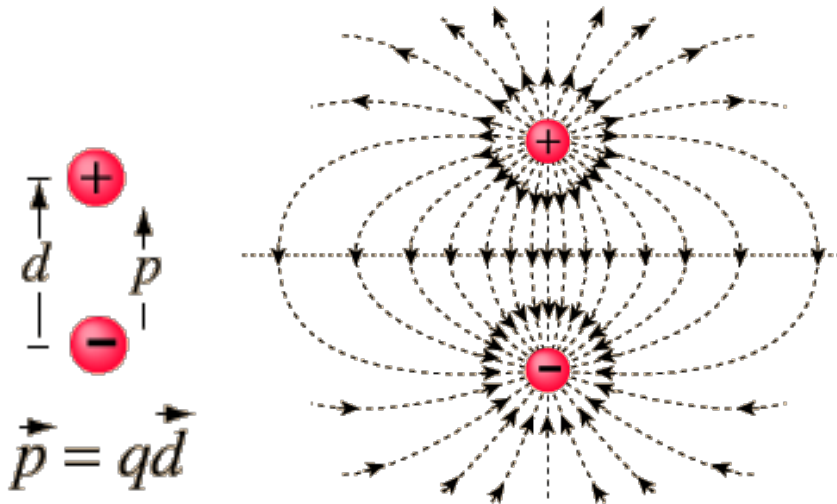
<https://www.youtube.com/watch?v=8tZOX7bBkvE>

<https://www.youtube.com/watch?v=YyofTn7tZjo>

<https://www.elisascience.org/multimedia/video/gravitationaluniverseisl3video>

Dipole Radiation

A spinning bar with positive charge on one end and negative on the other produces dipole electromagnetic radiation.



\mathbf{p} is the dipole moment

Quadrupole Radiation

For a pair of black holes with equal masses orbiting each other the **quadrupole moment** will be :

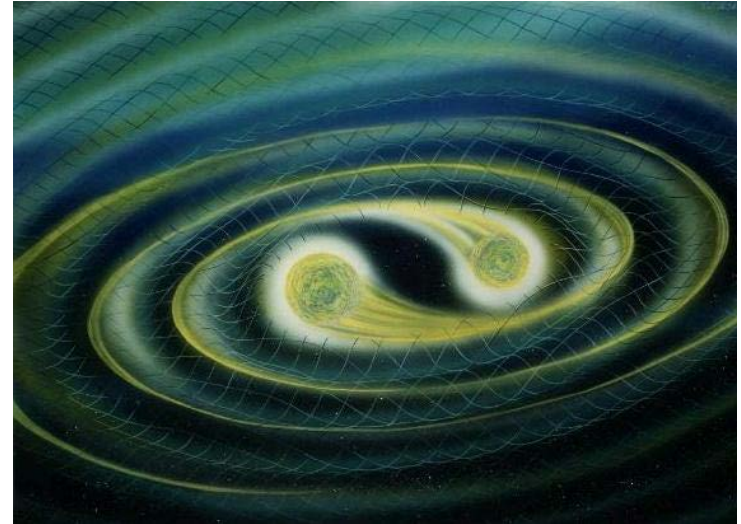
$$Q_{i,j} = M(3x_i x_j - \delta_{i,j})$$

$$\delta_{i,j} = 1 \text{ for } i = j \text{ and } \delta_{i,j} = 0 \text{ for } i \neq j$$

The indices i, j run over the coordinates.

We have placed the coordinate origin right between the black holes, and one black hole at unit distance along the x - axis.

As the system orbits the quadrupole of the black holes will change and the system will radiate gravitational waves. Energy lost in this way was indirectly detected in the Hulse-Taylor binary.



Gravitational Quadrupole Radiation

Experimental Tests of Frame-Dragging

The violet arrow displays frame-dragging of the Gravity Probe B gyroscopes by the Earth's spin, Ω_{L-T} , (~ 0.039 arcsec per year rotation of Gravity Probe B's spin axis around Earth's angular momentum \hat{J}).

The green arrow represents the geodetic precession, Ω_{geodetic} . Its theoretical value is ~ 6.6 arcsec per year about an axis orthogonal to the Gravity Probe B orbital plane.

Unfortunately due to unexpected large drifts of the gyroscopes' spin axes the geodetic precession was only measured to a precision of 1.5% (10^{-5} expected) and the error on the measurement of frame dragging Ω_{L-T} was relatively large.

