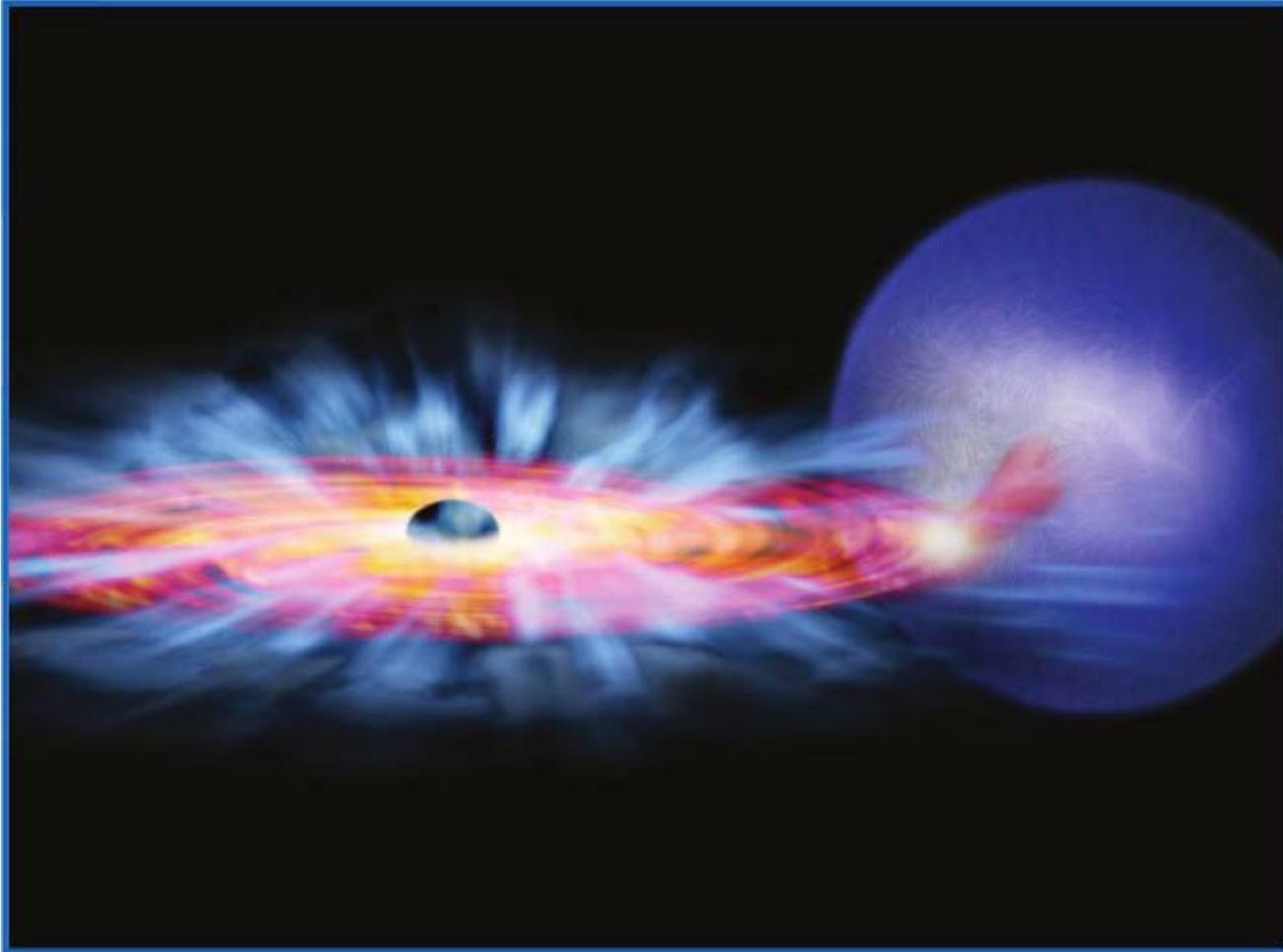


Black Holes



Special Relativity

Principles of Special Relativity:

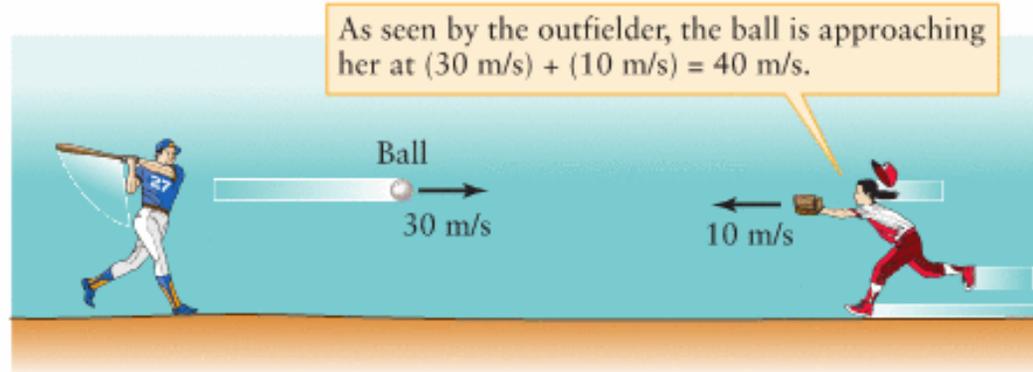
1. The laws of physics are the same for all inertial observers.

2. The speed of light is the same for all **inertial** observers regardless of the state of motion of the source.

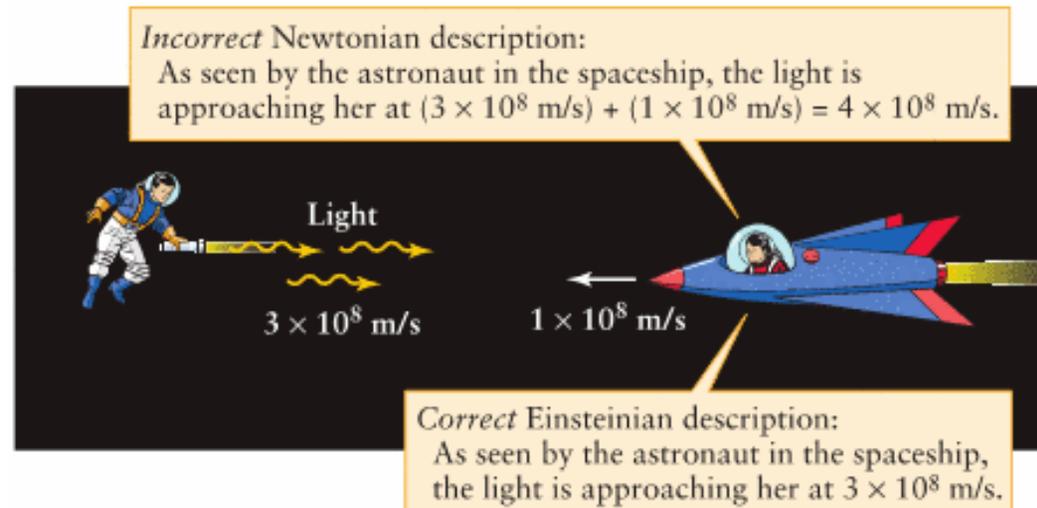
An **inertial observer** is one that is not accelerating.



Special Relativity



(a)



(b)

Special Relativity

The length you measure an object to have depends on how that object is moving; the faster it moves, the shorter its length along its direction of motion. This phenomenon is called **length contraction**.

$$L = L_0 \sqrt{1 - \left(\frac{v}{c}\right)^2}$$

L = observed length of object along direction of motion

L_0 = length of object at rest (proper length)

v = speed of object with respect to observer

c = speed of light

Special Relativity

A clock runs slower when observed by someone moving relative to the clock than someone not moving relative to the clock. This phenomenon is called **time dilation**.

$$T = \frac{T_0}{\sqrt{1 - \left(\frac{v}{c}\right)^2}}$$

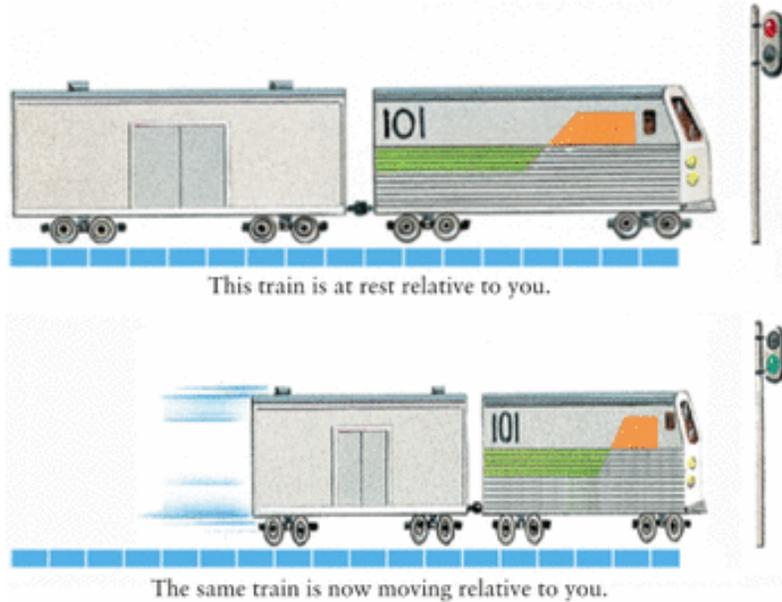
T = time interval measured by an observer moving relative to the phenomenon

T_0 = time interval measured by an observer not moving relative to the phenomenon

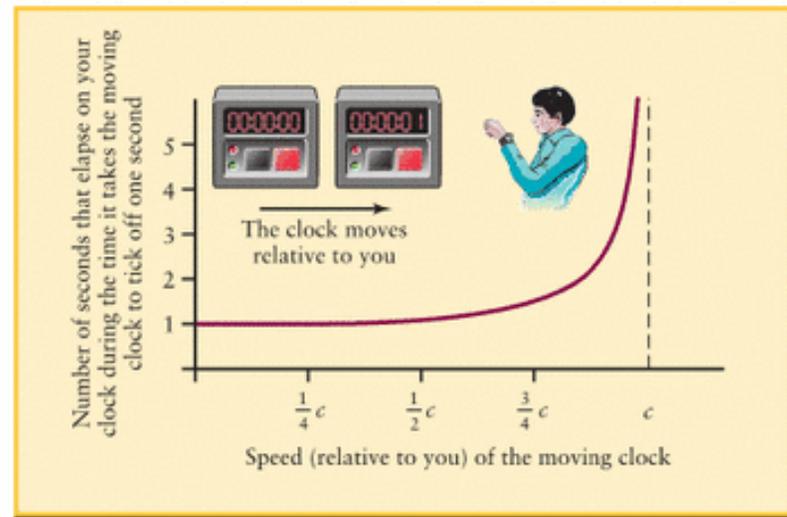
v = speed of phenomenon relative to observer

c = speed of light

Special Relativity



(a) Length contraction



(b) Time dilation

Special Relativity

Example 1: William is travelling in his spaceship at 98% of the speed of light relative to Alex. If William holds a 1 m ruler parallel to the direction of motion, how long is this ruler as measured by Alex?

Special Relativity

Example 2: When unstable particles called **muons** are produced in experiments on Earth, they **decay** into other particles **in an average time of 2.2×10^{-6} s**.

Muons are also produced by fast-moving protons from interstellar space when they collide with atoms in Earth's upper atmosphere. These **muons** typically move at 99.9% of the speed of light and are formed at an altitude of 10 km.

How long does it take for a muon to reach the Earth from 10 km (as observed from a non-moving observer)?

How long does it take for a muon to reach the Earth from 10 km (as observed from the muon)? Does the muon reach the Earth before decaying and why?

Special Relativity

Special relativity also predicts the famous relationship:

$$E = mc^2$$

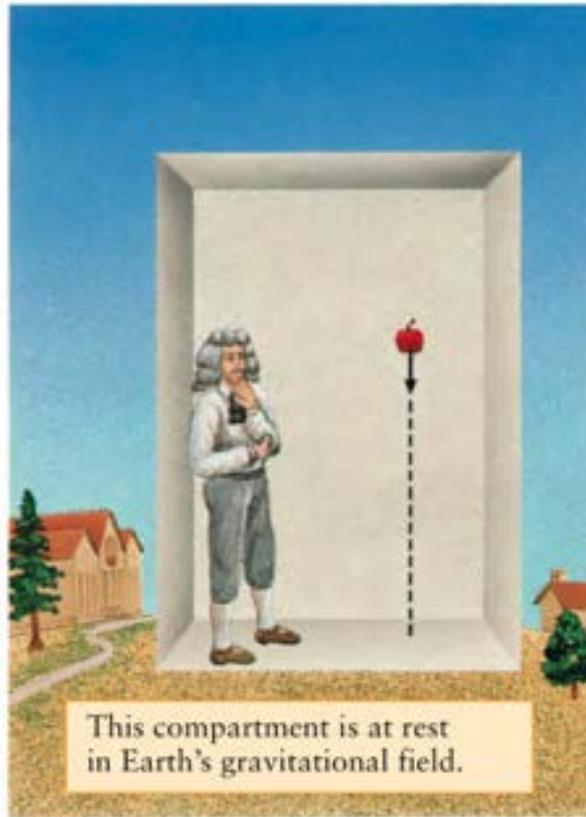
In fusion reactions mass is converted to energy and this released energy is what powers the stars.

General Theory of Relativity

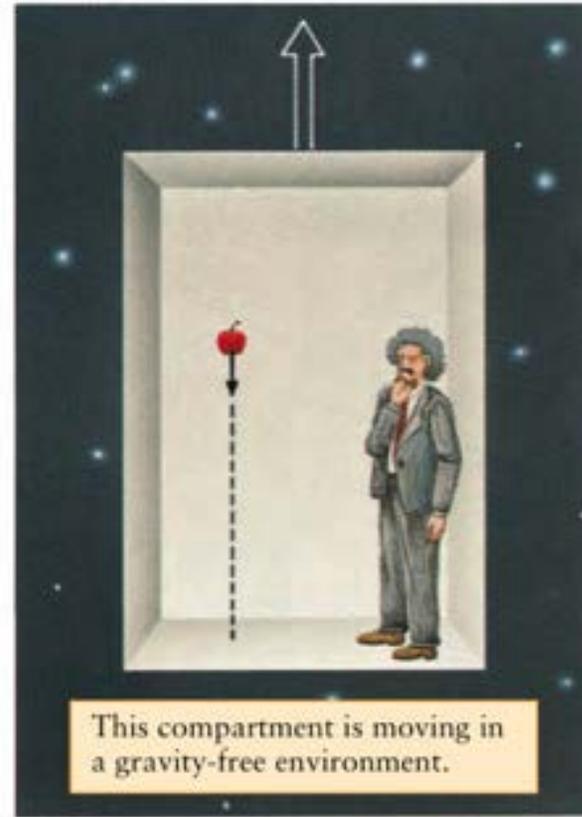
The **equivalence principle** asserts that you cannot distinguish between :

- (a) being at rest in a gravitational field and
 - (b) being accelerated upward in a gravity-free environment.
- This idea was an important step in Einstein's quest to develop the general theory of relativity.

General Theory of Relativity

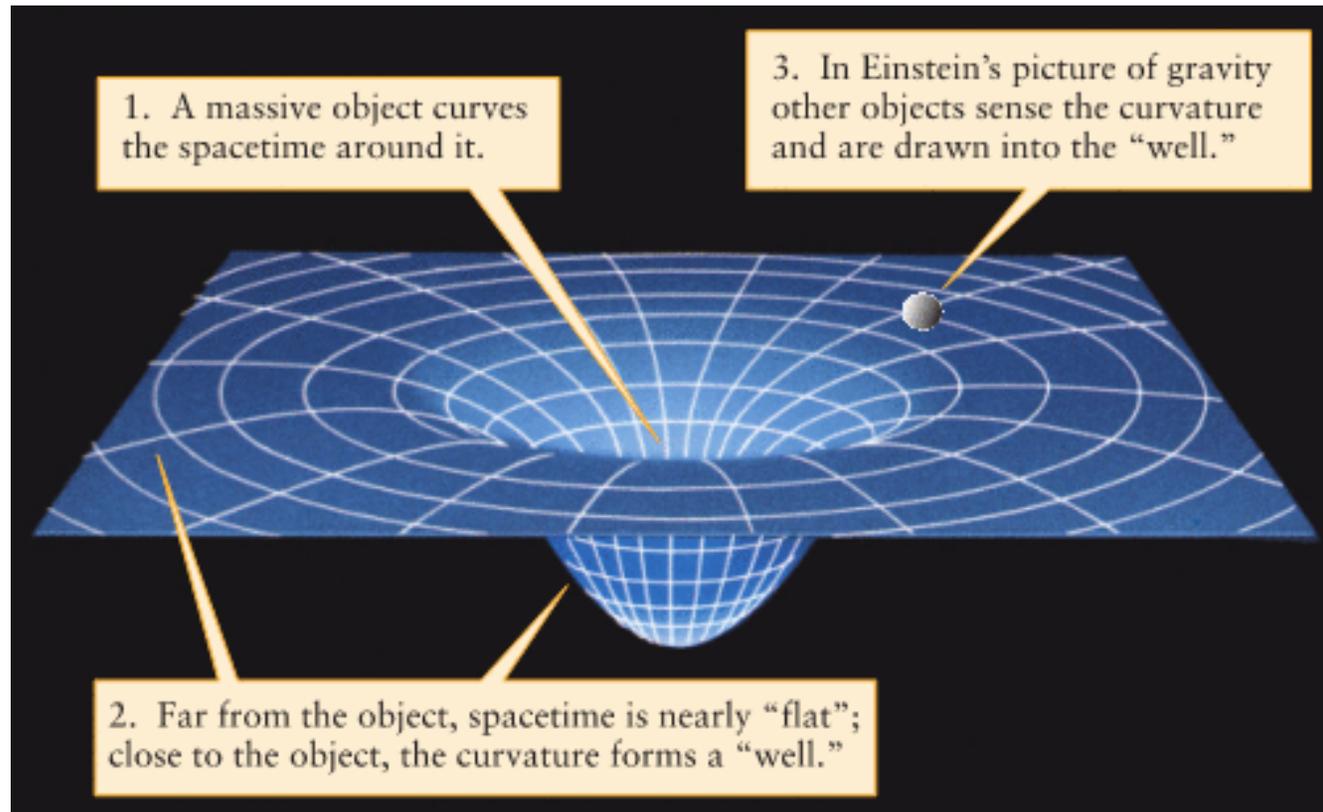


- (a) The apple hits the floor of the compartment because Earth's gravity accelerates the apple downward.



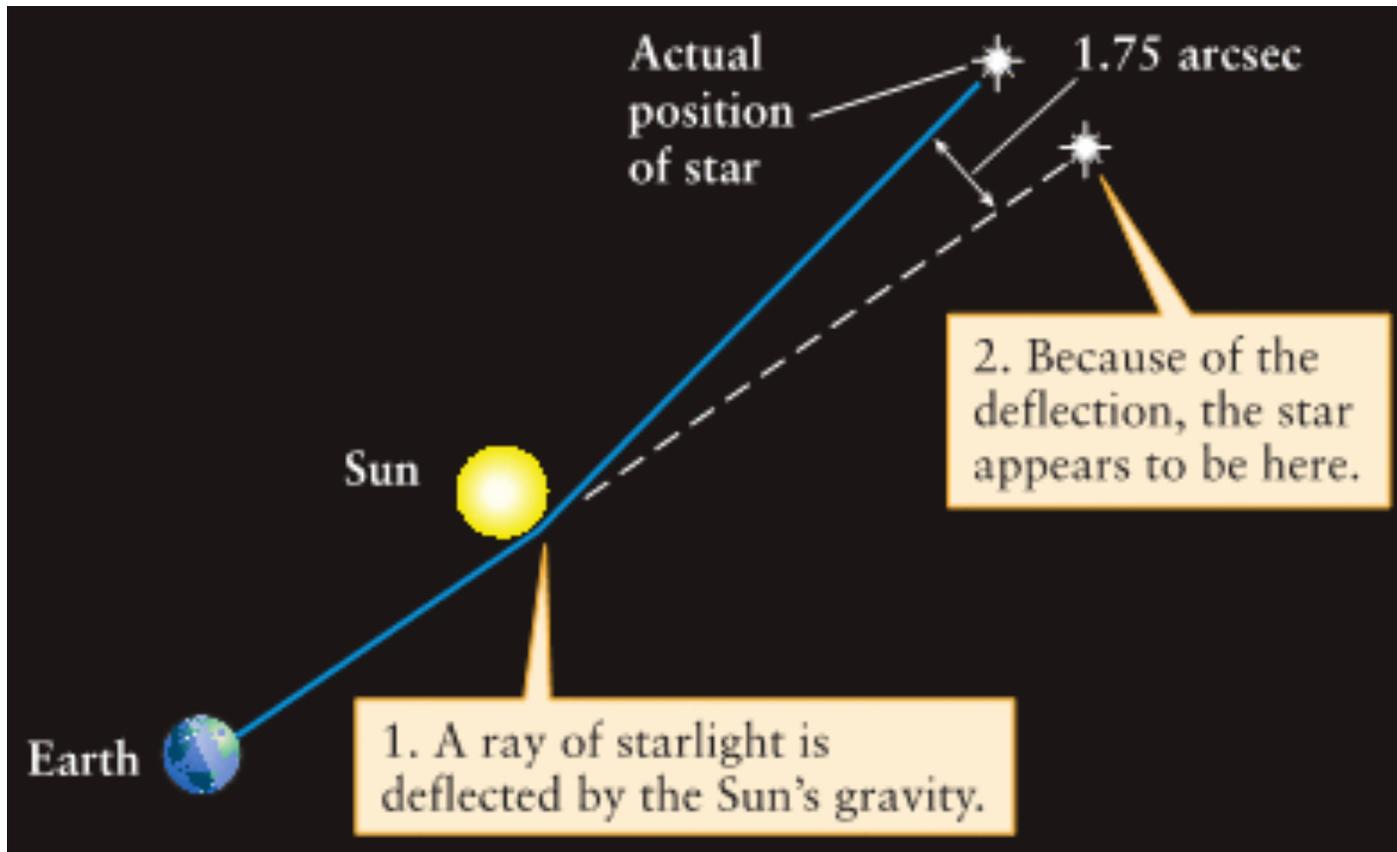
- (b) The apple hits the floor of the compartment because the compartment accelerates upward.

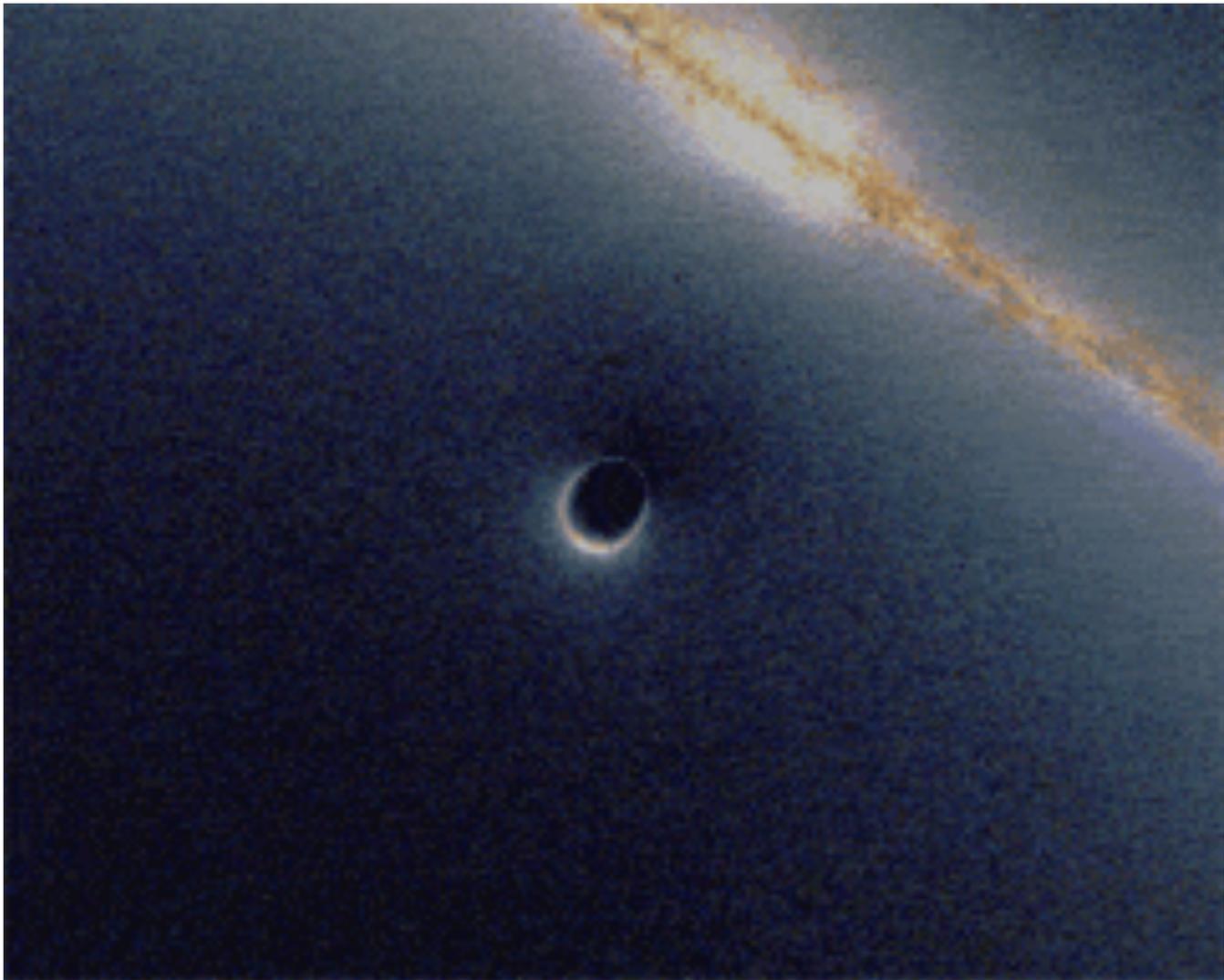
General Theory of Relativity



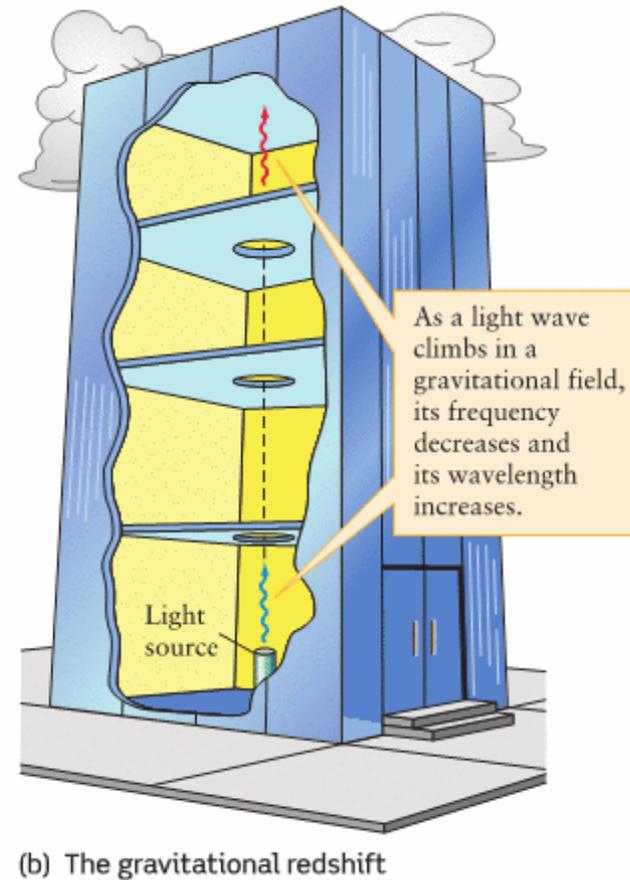
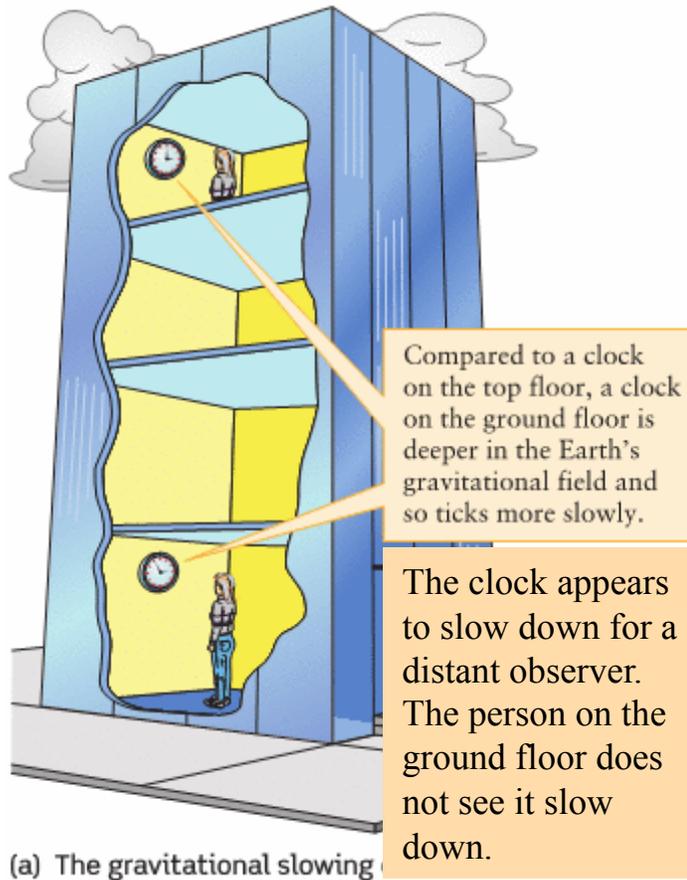
Einstein envisioned gravity as being caused by curvature of space. Above is a two dimensional analogy to help understand the curvature of four-dimensional spacetime.

Testing the General Theory of Relativity



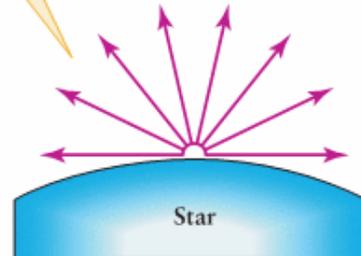


Gravitational Slowing of Time and Gravitational Redshift



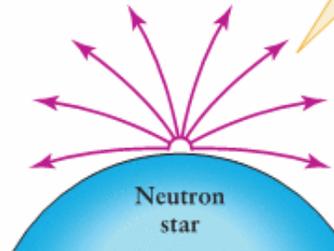
Relativity Predicts Black Holes

1. A supergiant star has relatively weak gravity, so emitted photons travel in essentially straight lines.



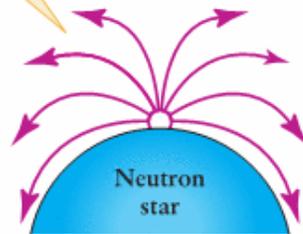
(a)

2. As the star collapses into a neutron star, the surface gravity becomes stronger and photons follow curved paths.



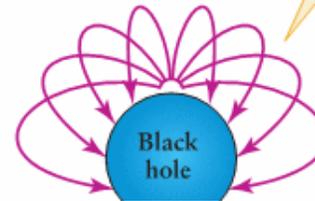
(b)

3. Continued collapse intensifies the surface gravity, and so photons follow paths more sharply curved.



(c)

4. When the star shrinks past a critical size, it becomes a black hole: Photons follow paths that curve back into the black hole so no light escapes.



(d)

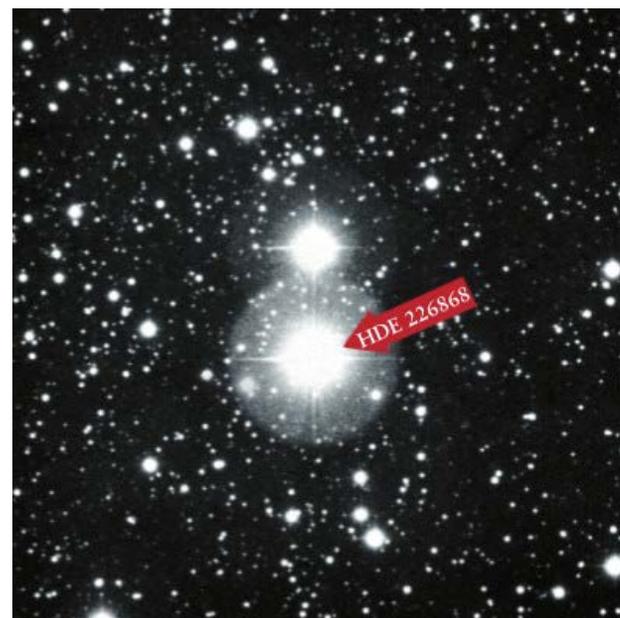
When a star becomes a black hole, not even photons emitted directly upward from the surface can escape; they undergo an infinite gravitational redshift and disappear.

Black Hole Candidates

Cygnus X-1 is thought to be a black hole in a binary system with a companion supergiant B0 star. The X-ray emission of Cygnus X-1 changes significantly within 0.01 sec. This places a limit on the size of the object.

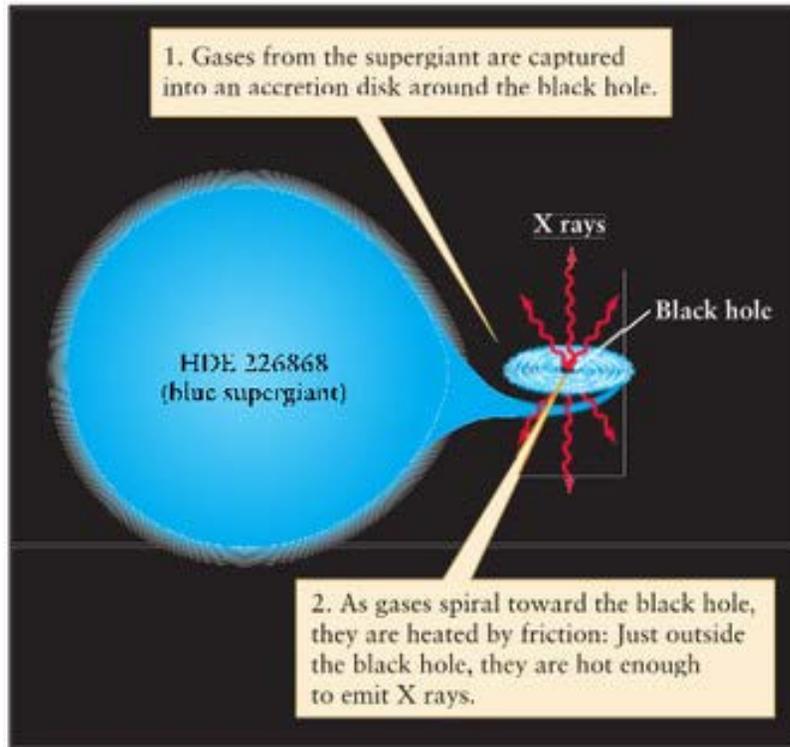
The shortest variation time in the flux of an object of size D is the time that light takes to travel across the object (t_{lc}).

$$\begin{aligned} t_{\text{variation}} > t_{lc} = D/c &\rightarrow D < ct_{\text{variation}} \rightarrow \\ \rightarrow D < 3 \times 10^5 \text{ km/sec} \times 0.01 \text{ sec} \\ \rightarrow D < 3,000 \text{ km} \end{aligned}$$



HDE 226868 is a B0 star with an estimated mass of $30 M_{\odot}$ and the other object in the binary is estimated to have a mass of $\sim 7M_{\odot}$ implying that it is a black hole.

Black Holes



(a) A schematic diagram of Cygnus X-1

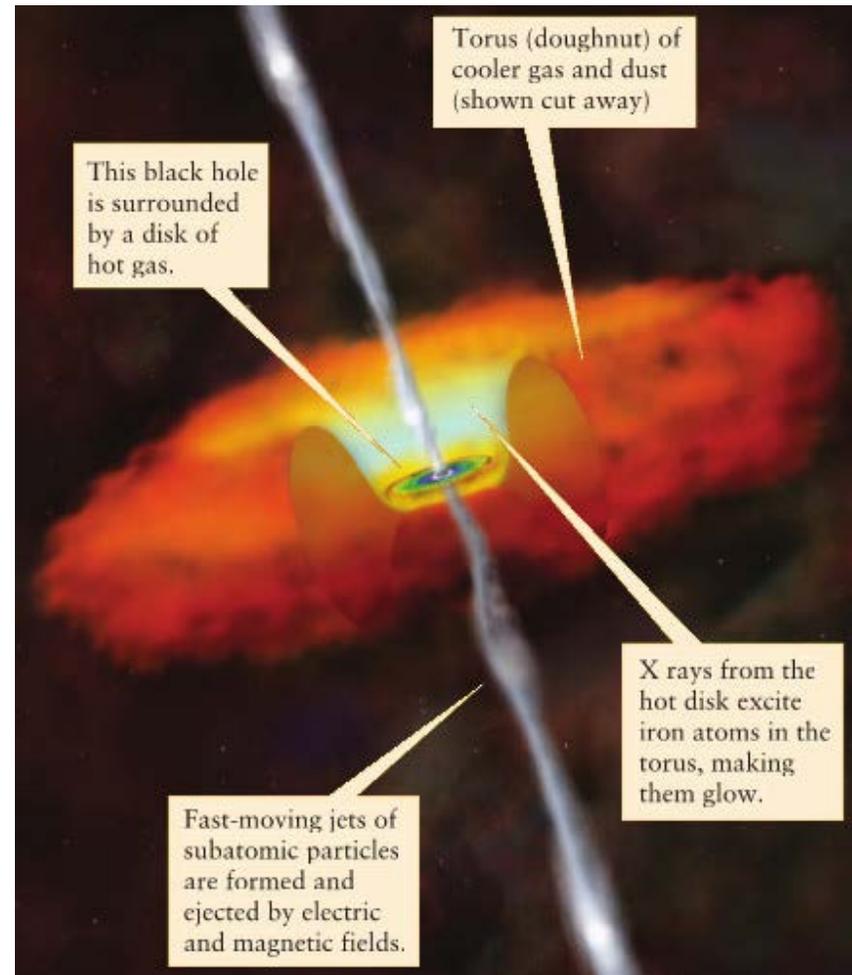


(b) An artist's impression of Cygnus X-1

Jets from Stellar Mass Black Holes

Jets of charged particles moving at near the speed of light are observed in stellar mass black hole systems.

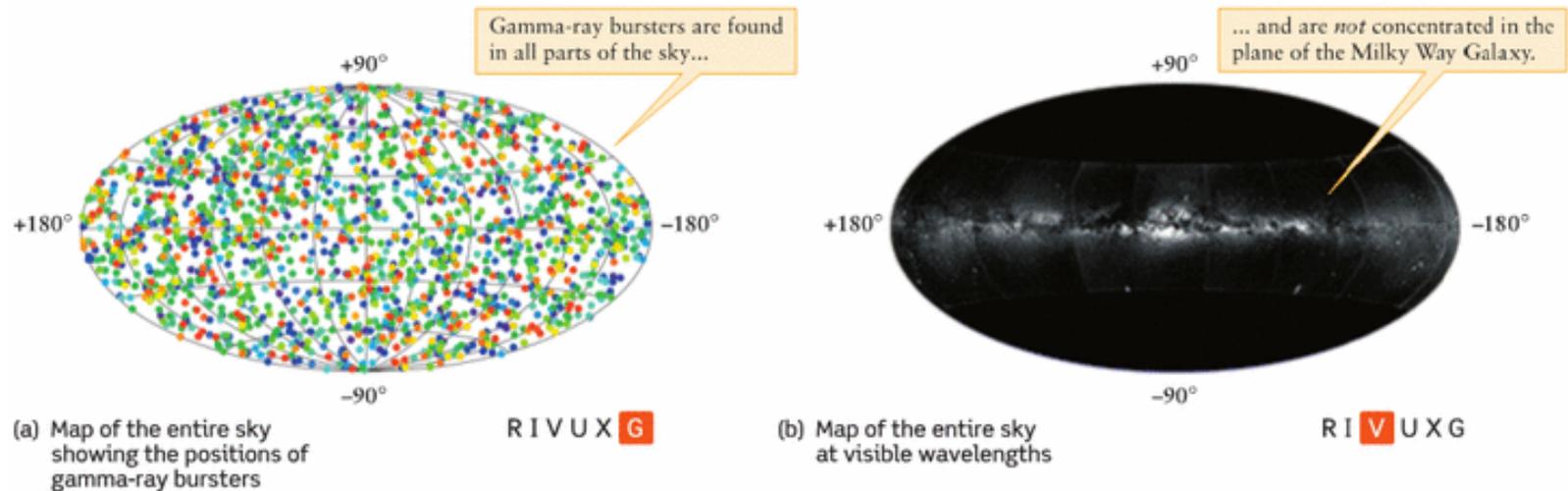
It is thought that magnetic fields collimate material from the accretion disc into these narrow beams.



Forming a Stellar Mass Black Hole in a Binary

- (1) A black hole is formed in a binary system in which one of the stars evolves to explode in a core collapse Type II SN and the remaining burnt-out core has a mass of more than $3 M_{\odot}$ after the explosion. Such a massive star cannot be supported by electron degeneracy pressure ($1.4 M_{\odot}$ limit) or by neutron degeneracy pressure ($\sim 3 M_{\odot}$ limit).
- (2) A black hole may be formed in a binary system with a WD or NS. If mass transfer onto the WD or NS from the companion giant star makes the mass of the WD or NS become larger than $3M_{\odot}$ it will collapse to become a BH.
- (3) A black hole may be formed in the merger of two neutron stars in a binary system.

Gamma Ray Bursts



Gamma-ray bursts were discovered in the late 1960s by the Vela satellites. They fall into two types. **Long bursts** ~ 2 - 1000 sec duration and **short bursts** ~ 0.01 sec - 2 sec duration (contain lower energy photons than long bursts)

Their cosmic origin was confirmed with the measurement of their redshift. The spectra of the **afterglows** are consistent with Type Ib or Ic supernova spectra.

Collapsar Model of Gamma Ray Bursts

The **collapsar model** proposes that a **long gamma ray burst** occurs during a core collapse supernova (Type Ic) of a $\sim 30 M_{\odot}$ star that is spinning rapidly.

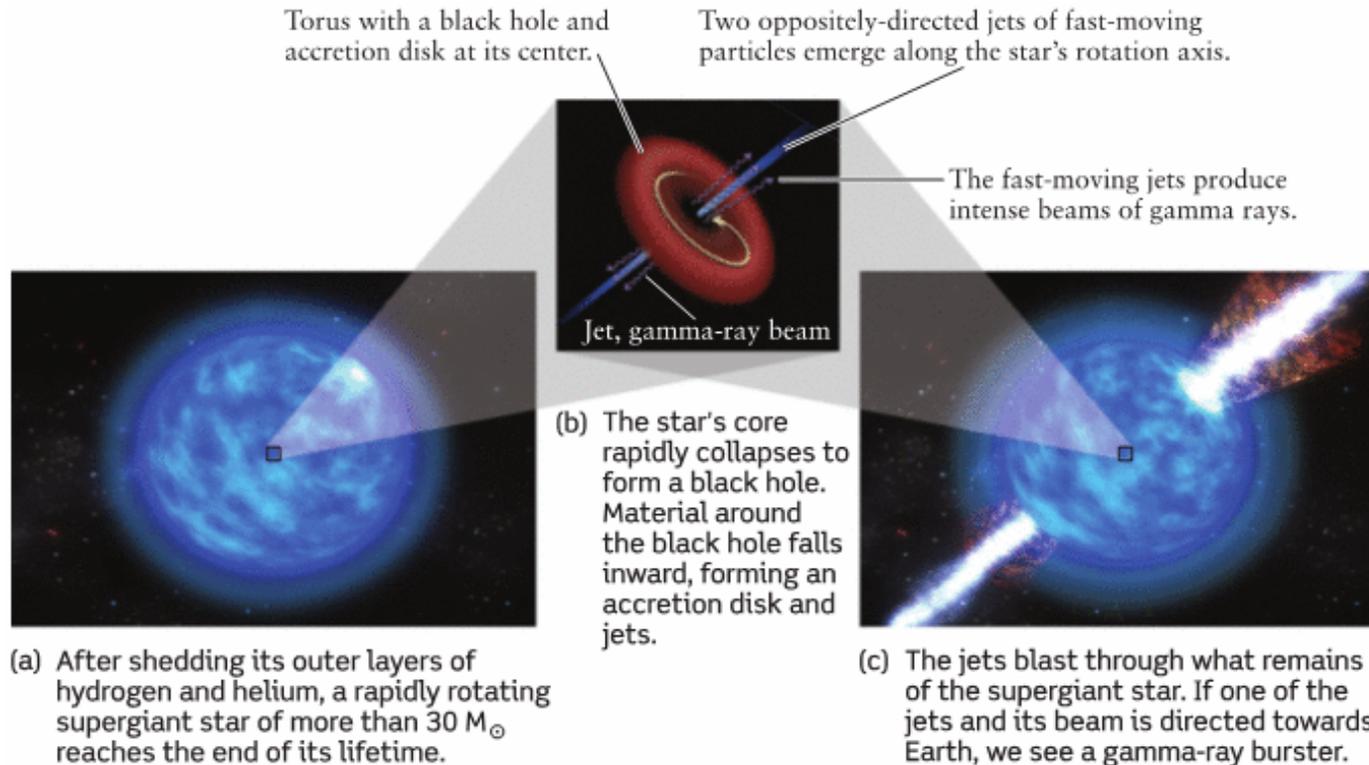
In this model **the core of the star collapses** to form a BH. The material just around the BH forms an accretion disk that is drawn into the BH. The magnetic field of this accretion disk forms jets of charged particles that break through the outer layers of the star.

(Key point: in a Type Ic supernova the outer **H and He layers have been blown away** so its easier for the jets to break through the star)

The **energetic particles in the jet produce gamma rays**. If a jet is pointed towards the Earth we see a gamma ray burst.

The accretion disk is sucked into the BH within a few seconds and the gamma rays burst ends.

Collapsar Model



Beamed radiation: A typical Type Ic supernova releases 10^{46} Joules of energy (0.03% goes into light and the rest mostly into neutrinos)

If we were to assume that all the energy released in a gamma ray burst was distributed uniformly the total energy would be $\sim 3 \times 10^{47}$ Joules but if the energy is beamed then the total energy is less and close to a normal Type Ic.

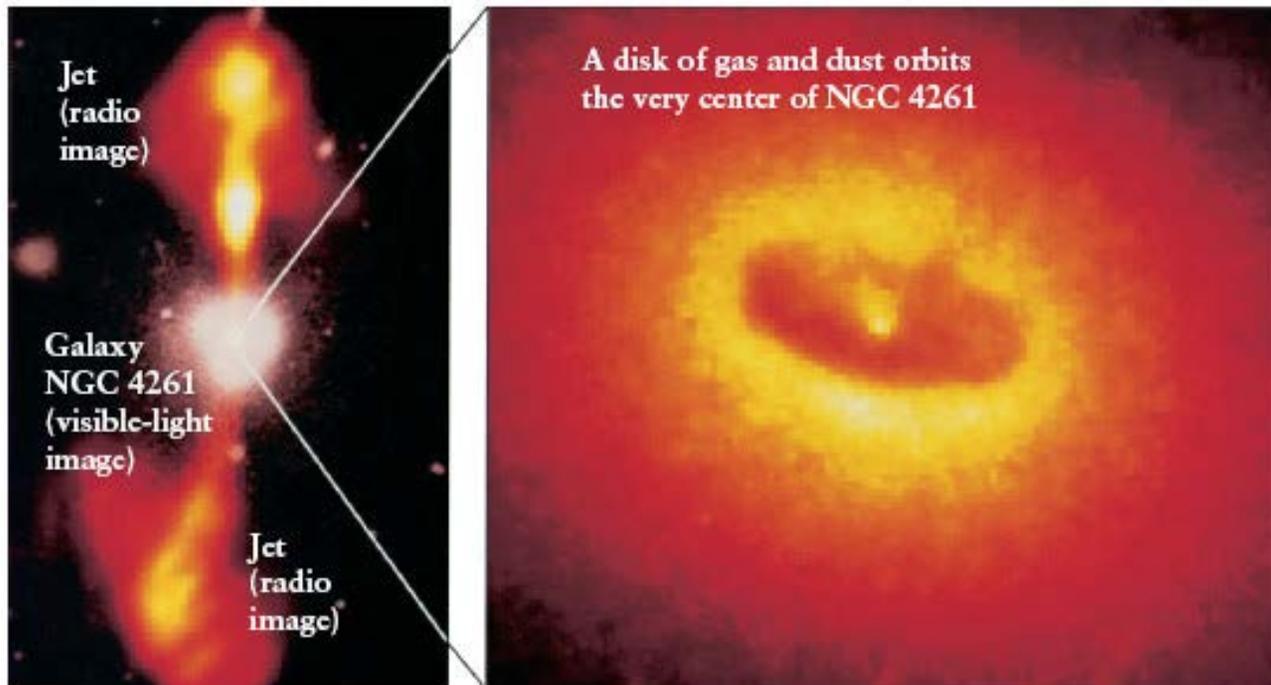
Supermassive Black Holes

Astronomers have discovered black holes with masses ranging from 10^6 to $10^{10} M_{\odot}$ in the centers of most galaxies. It is not clear how **supermassive black holes** (SMBH) form and how they grow so large so early.

SMBH's are usually detected from radio, optical, UV and X-ray emission originating from material surrounding the black hole.

Another way of detecting SMBH's is from their gravitational influence on material in their near vicinity.

SMBH in the center of galaxy NGC 4261



(a) Galaxy NGC 4261

(b) Evidence for a supermassive black hole in NGC 4261

R I V U X G

R I V U X G

(a) A radio image of jets emanating from the SMBH in NGC 4261. A visible image of the accretion disk surrounding the SMBH is superimposed.

(b) A Hubble Space Telescope image of the center of NGC 4261 shows a disk of gas and dust orbiting the SMBH.

Non Rotating Black Holes

The **event horizon** of a BH is a sphere surrounding it where the escape speed is equal to the speed of light. Anything that crosses the event horizon cannot escape and exit the black hole.

For a non-rotating BH the distance from the center to the event horizon is called the **Schwarzschild radius**.

The star's entire mass is crushed to a single point, known as the singularity, at the center of the black hole.



$$R_{Sch} = \frac{2GM_{BH}}{c^2}$$

R_{Sch} = Schwarzschild radius

$G = 6.67 \times 10^{-11} \text{ N m}^2/\text{kg}^2$

$c = 3 \times 10^8 \text{ m/s}$

M_{BH} = mass of black hole

Non Rotating Black Holes

Example: What is the Schwarzschild radius of a BH with a mass of $M_{\text{BH}} = M_{\odot} \sim 2 \times 10^{30} \text{ kg}$?

The Schwarzschild radius is directly proportional to the mass of the black hole.

Without using a calculator estimate the Schwarzschild radius of a billion solar mass black hole.



$$R_{\text{Sch}} = \frac{2GM_{\text{BH}}}{c^2}$$

R_{Sch} = Schwarzschild radius

$G = 6.67 \times 10^{-11} \text{ N m}^2/\text{kg}^2$

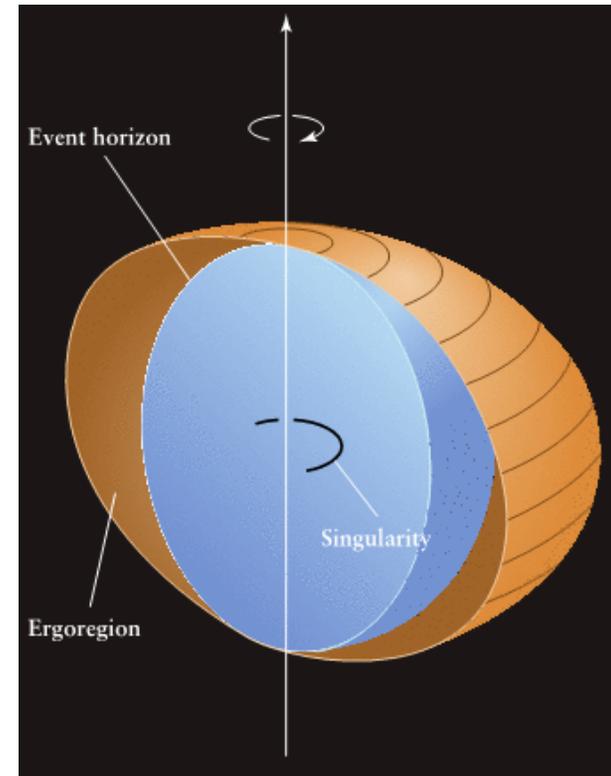
$c = 3 \times 10^8 \text{ m/s}$

M_{BH} = mass of black hole

Rotating Black Holes

When the matter that collapses to form a black hole is rotating, that matter does not compress to a point. Instead, it collapses into a ring-shaped singularity located between the center of the hole and the event horizon.

The structure of such rotating black holes was first worked out in 1963 by the mathematician Roy Kerr.



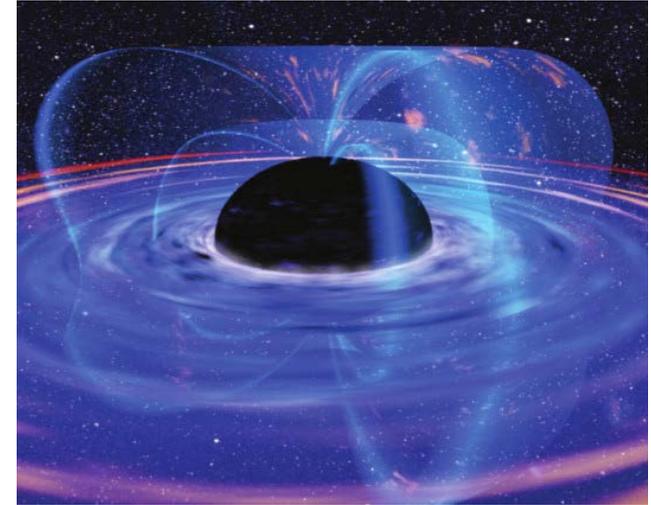
Outside the spherical event horizon is the doughnut-shaped ergoregion, where the dragging of spacetime around the hole is so severe that nothing can remain at rest.

Properties of Black Holes

1. Mass: One can estimate the mass of a BH by measuring the size and period of an object that orbits the BH and using Newton's form of Kepler's third law.

2. Electric Charge: BH's are expected to have very little electric charge.

3. Angular Momentum: Conservation of angular momentum implies that when a rotating star collapses to form a BH the resulting BH will be spinning much faster since the material is crushed to a much smaller radius.



An artist's impression of the accretion disk around the supermassive BH at the heart of the galaxy MCG-6-15-30. The arching magnetic field allows the accretion disk to extract energy and angular momentum from the black hole.

Falling Into a Black Hole

Imagine sending a probe into a black hole equipped with a camera to send you images of what it sees.

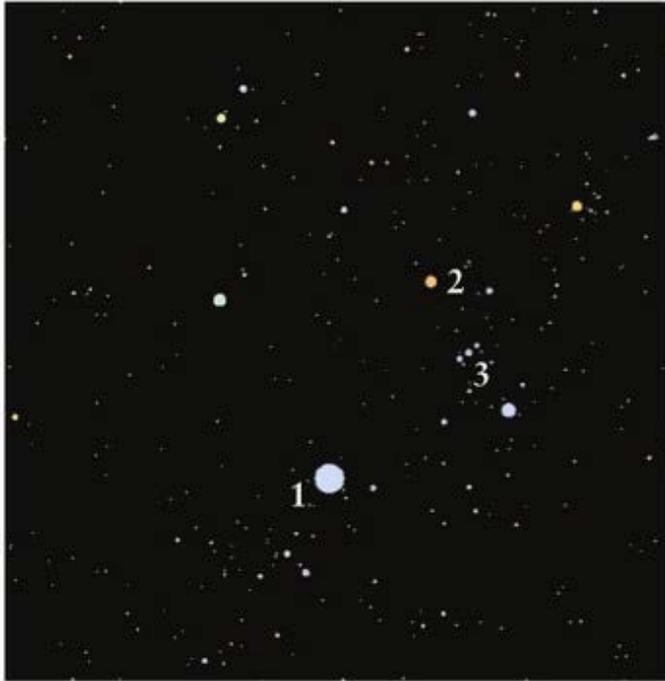
-As the probe approaches the BH the severe bending of light will make stars that are almost behind the BH form multiple images and stars exactly behind form rings.

-The probe itself will initially appear to be speeding up but as it approaches the BH time dilation will make it appear moving slower and eventually will appear not to be moving.

-The color of the probe will appear to increase in wavelength due to gravitational redshift.

-As the probe gets closer the strong tidal forces stretch the probe along the line pointing towards the center and also make it squeezed together along the perpendicular direction.

Black Holes

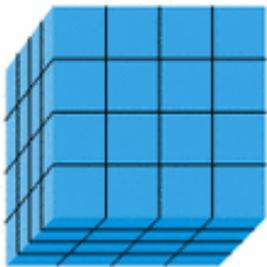


(a) Looking directly toward the black hole from a distance of 1000 Schwarzschild radii: Note positions of stars 1, 2, and 3.



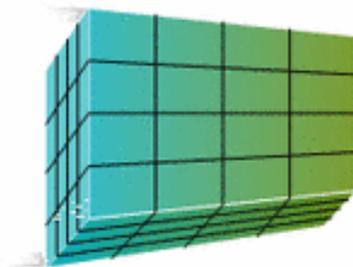
(b) Looking directly toward the black hole from a distance of 10 Schwarzschild radii: Light bending causes multiple images.

Probe far from black hole

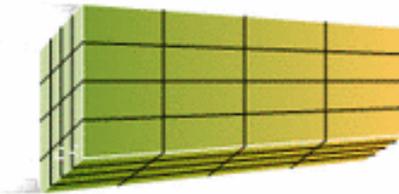


(a)

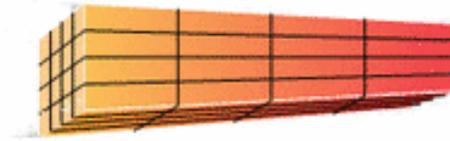
Probe approaching black hole



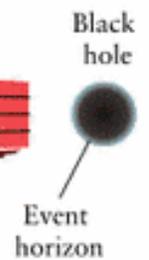
(b)

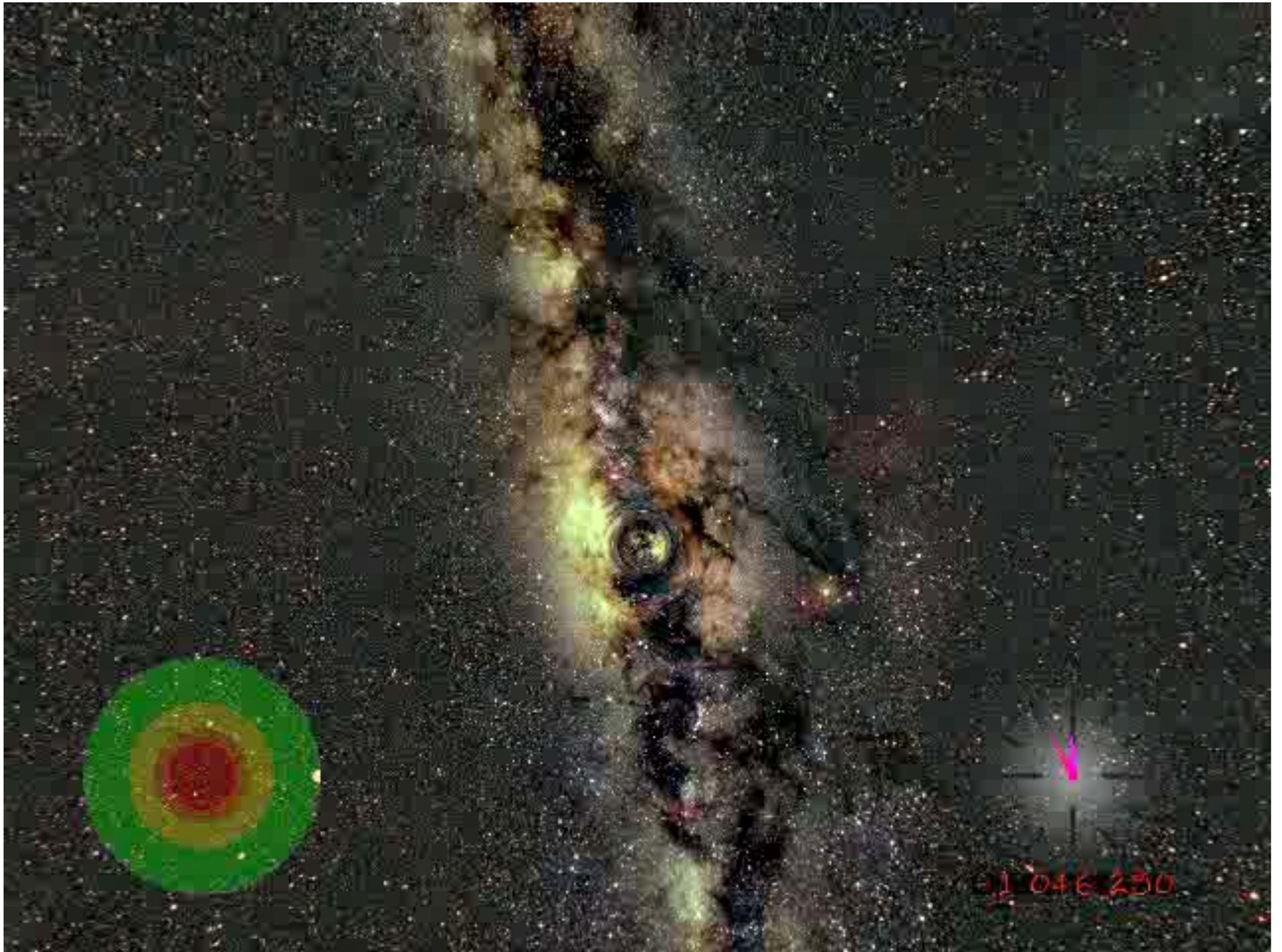


(c)

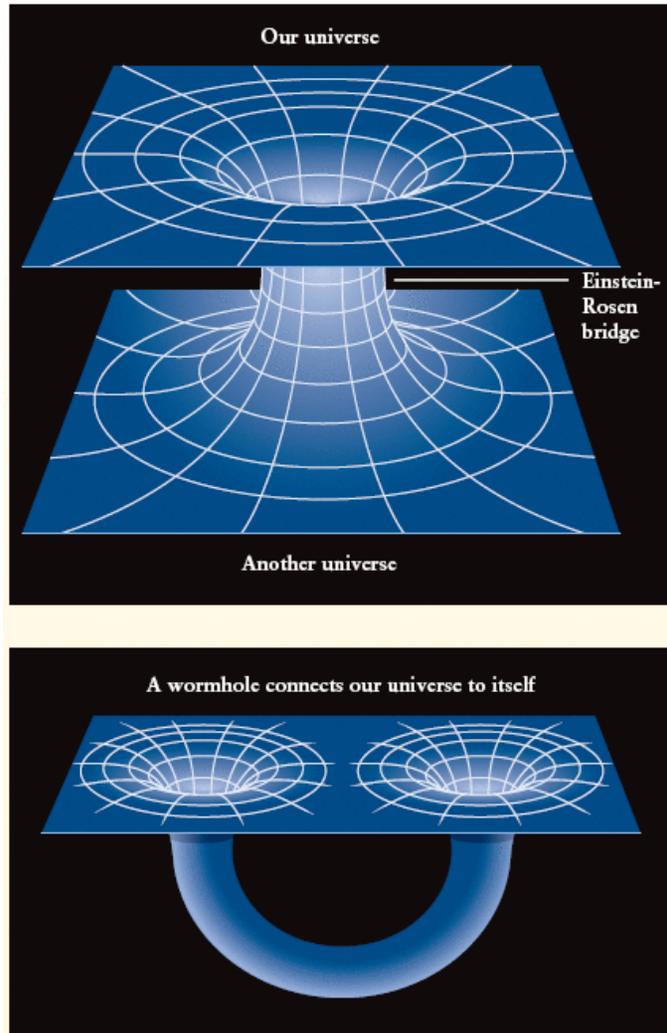


(d)





Black Holes



Black Holes

Particles and antiparticles are constantly being created and destroyed out of vacuum fluctuations.

If a pair of **virtual particles** is created very close to the event horizon of a BH it is possible that one gets pulled into the BH and the other one escapes.

Since the particles were created from the Black Hole's gravitational energy the escaping particle has carried away some of the BH's mass.

