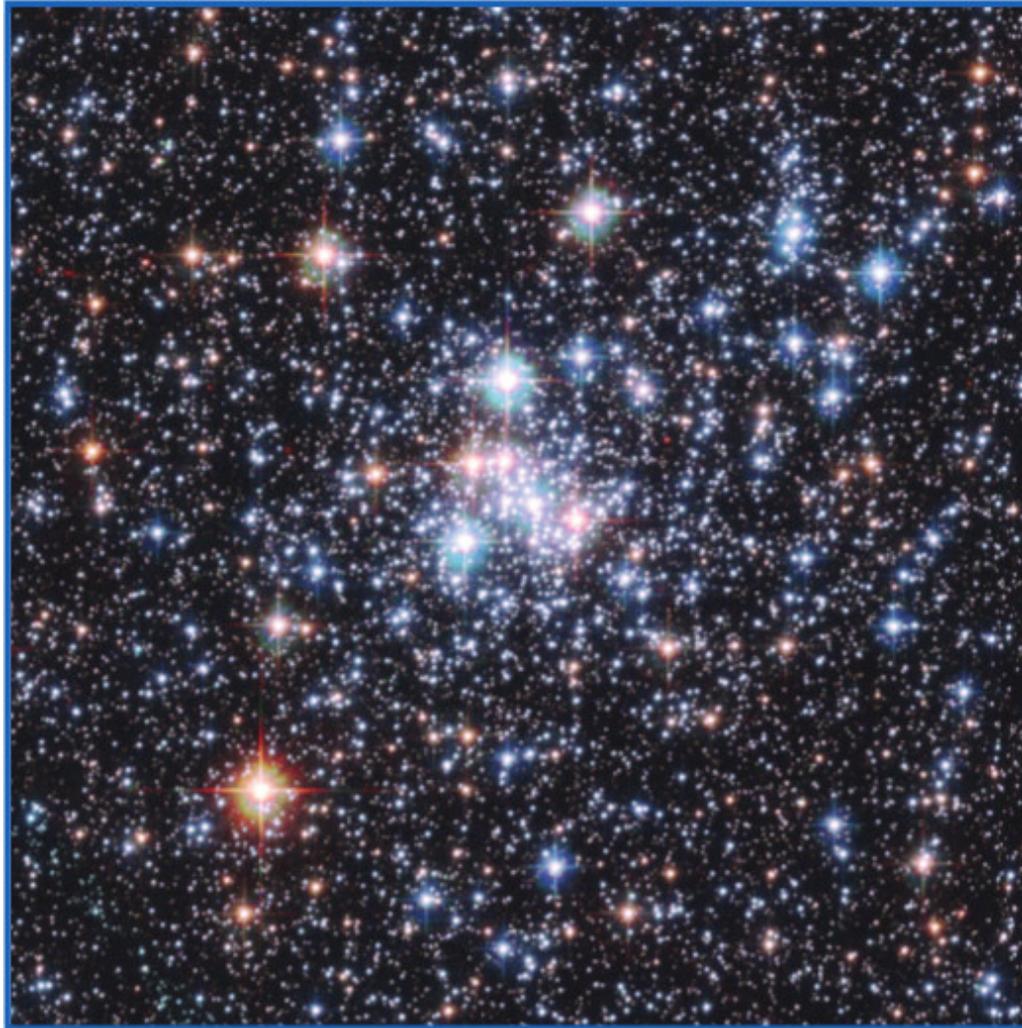
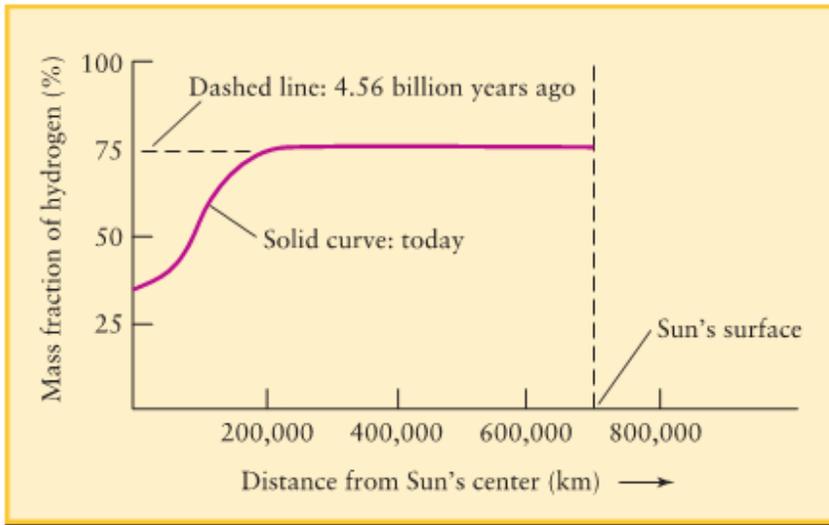


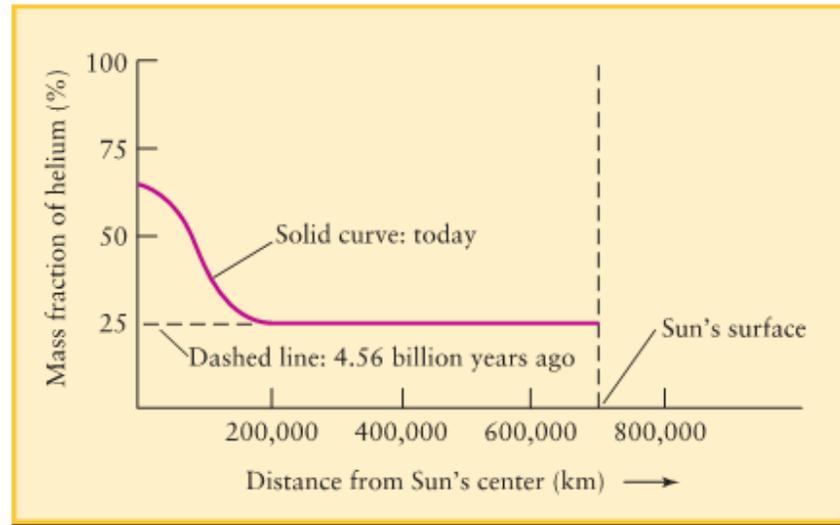
Stellar Evolution: On and After the Main Sequence



Stellar Evolution: On and After the Main Sequence



(a) Hydrogen in the Sun's interior



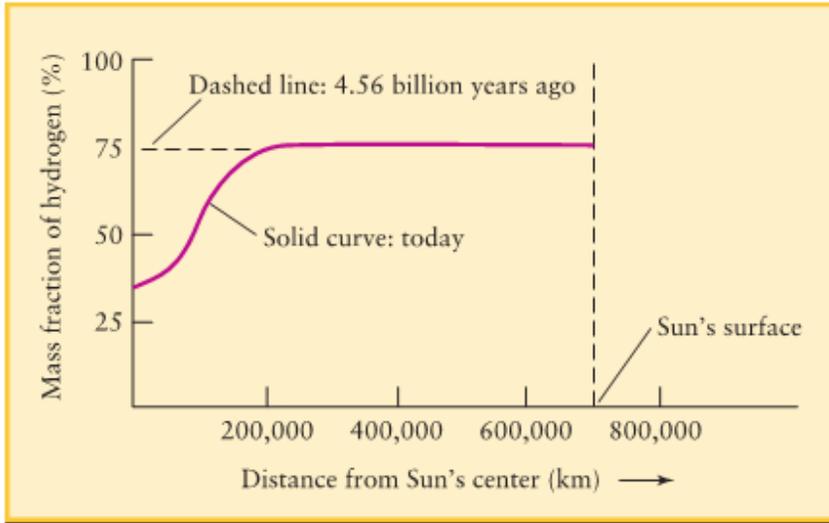
(b) Helium in the Sun's interior

First we consider the case of $M_{\text{star}} > 0.4 M_{\odot}$

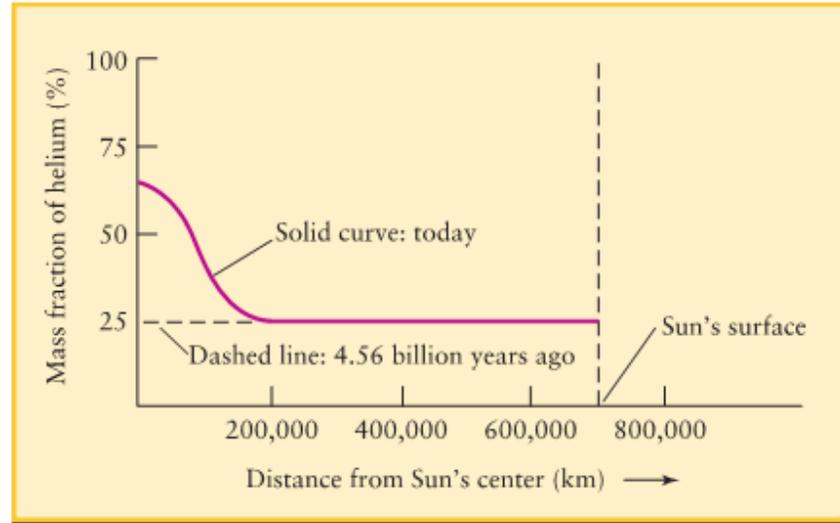
Initially when the star begins on the main sequence the mass fraction of hydrogen is about 74% by mass throughout the star, helium is about 25%, and the rest is metals.

As H fuses into He the abundance of H in the core decreases and the abundance of He in the core increases with time.

Stellar Evolution: On and After the Main Sequence



(a) Hydrogen in the Sun's interior



(b) Helium in the Sun's interior

Some definitions:

main-sequence lifetime: the time a star will spend on the main sequence fusing hydrogen to helium. What is the main sequence lifetime of the sun?

zero-age main-sequence star: A newly formed star that has just arrived on the main sequence.

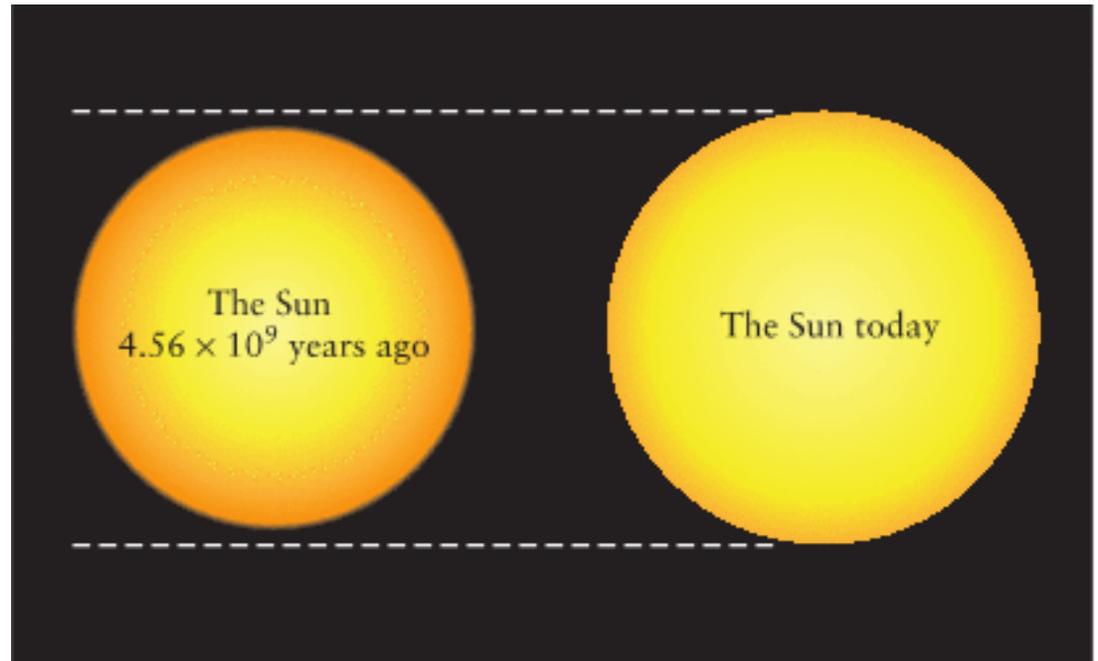
Stellar Evolution: On and After the Main Sequence

After 4.56 billion years on the main sequence the Sun has:

$$\Delta R_{\odot} = 6\% R_{\odot} \text{ (increase)}$$

$$\Delta T_{\odot} = 5\% T_{\odot} \text{ (increase)}$$

$$\Delta L_{\odot} = 40\% L_{\odot}$$



Why does a star's core slowly contract as it evolves on the main sequence?

Why does a star's outer layers expand as it evolves on the main sequence?

Stellar Evolution: On and After the Main Sequence

Table 19-1 Approximate Main-Sequence Lifetimes

Mass (M_{\odot})	Surface temperature (K)	Spectral class	Luminosity (L_{\odot})	Main-sequence lifetime (10^6 years)
25	35,000	O	80,000	4
15	30,000	B	10,000	15
3	11,000	A	60	800
1.5	7000	F	5	4500
1.0	6000	G	1	12,000
0.75	5000	K	0.5	25,000
0.50	4000	M	0.03	700,000

The main-sequence lifetimes were estimated using the relationship $t \propto 1/M^{2.5}$ (see Box 19-2).

How long will a star remain on the main sequence?

$E = fMc^2$, where f is the fraction of the star's mass that's converted into energy

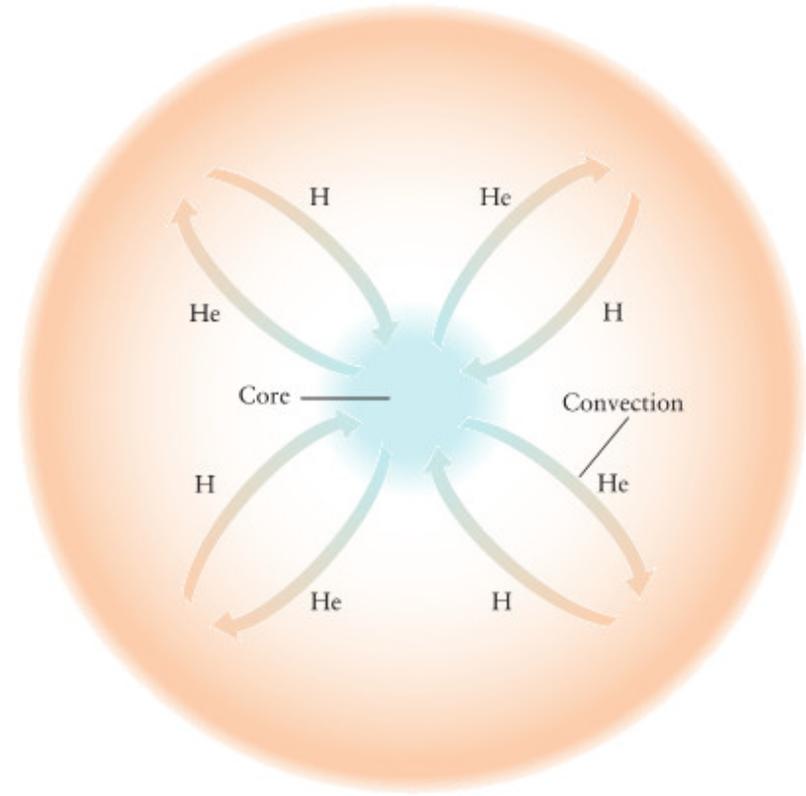
$$L = \frac{E}{t} \Rightarrow t = \frac{E}{L} = \frac{fMc^2}{L} \propto \frac{M}{M^{3.5}} = \frac{1}{M^{2.5}}$$

$$t_{star} = t_{solar} \left(\frac{M_{solar}}{M_{star}} \right)^{2.5}, \quad t_{solar} = 1.2 \times 10^{10} \text{ years}$$

Stellar Evolution: On and After the Main Sequence

Case of $M_{\text{star}} < 0.4 M_{\odot}$

These stars are called **red dwarfs** because they are less massive than the sun and they are red in color due to their low temperature.



85 % of the stars in the Milky Way are M class red dwarfs. **Energy in red dwarfs is transported by convection** from the core to the outer layers and therefore they do not build up He only in the core as do stars with $> 0.4 M_{\odot}$.

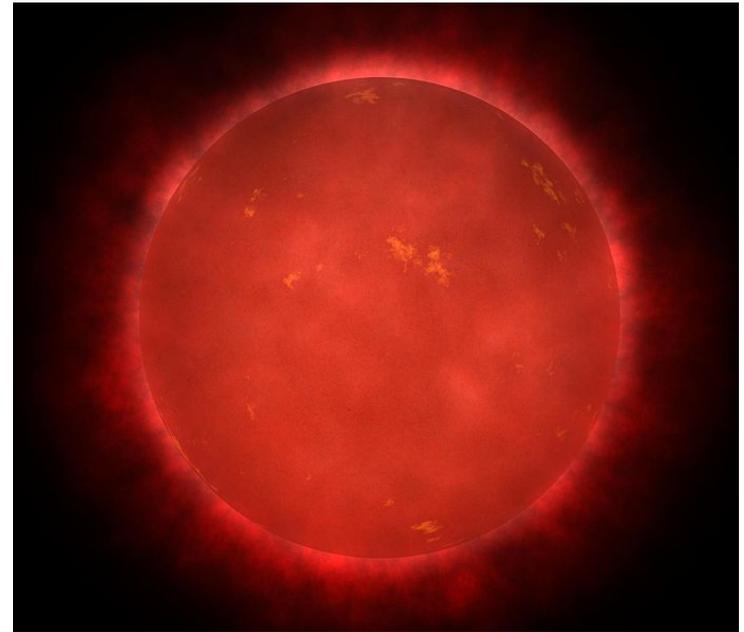
Stellar Evolution: On and After the Main Sequence

What happens when the H runs out?

$M < 0.4M_{\odot}$ In a red dwarf when the H runs out the temperature and density are not high enough to initiate fusion of He.

After the H fusion cycle ends in a red dwarf it radiates its energy away, slowly cools and shrinks in size.

Calculations indicate that it takes \sim 100 billion years for a red dwarf to convert all of its H to He. Why haven't we detected such an object yet?



Artistic presentation of a red dwarf star.

Stellar Evolution: On and After the Main Sequence

What happens when the H runs out in a $M > 0.4M_{\odot}$?

WHAT HAPPENS IN THE STARS CORE?

In a star like our sun when H runs out in the core **H fusion continues in a shell around the core**. Initially the star's core starts to cool and the pressure there starts to decrease.

The core cannot support the material above it so it begins to shrink and is compressed from the weight of the outer layers. When the gas in the core becomes compressed its temperature again begins to rise (remember example with bike pump).

The increase in temperature increases the rate of H fusion reactions in the thin H shell that surrounds the shell while there is no core H fusion.

Stellar Evolution: On and After the Main Sequence

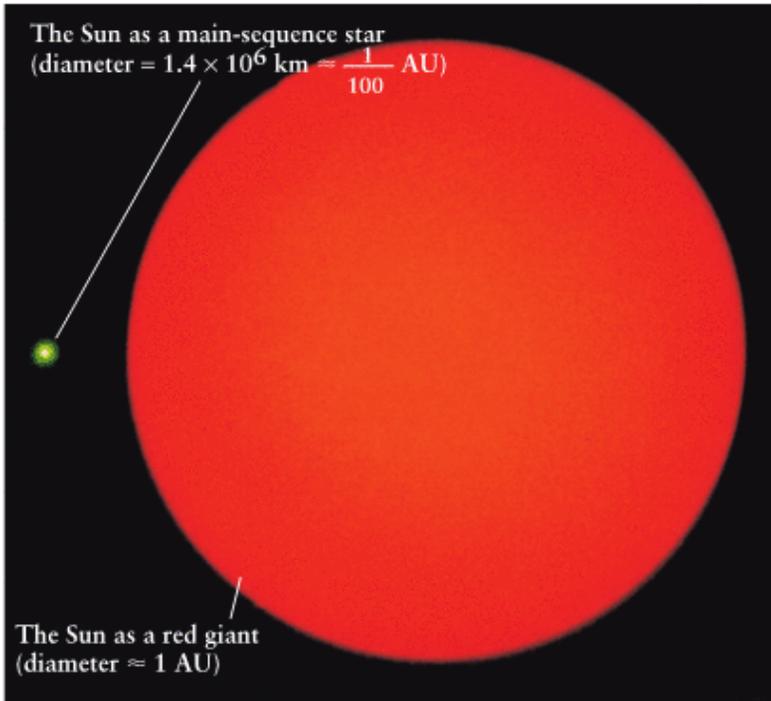
What happens when the H runs out in a $M > 0.4M_{\odot}$?

WHAT HAPPENS IN THE STARS OUTER LAYERS?

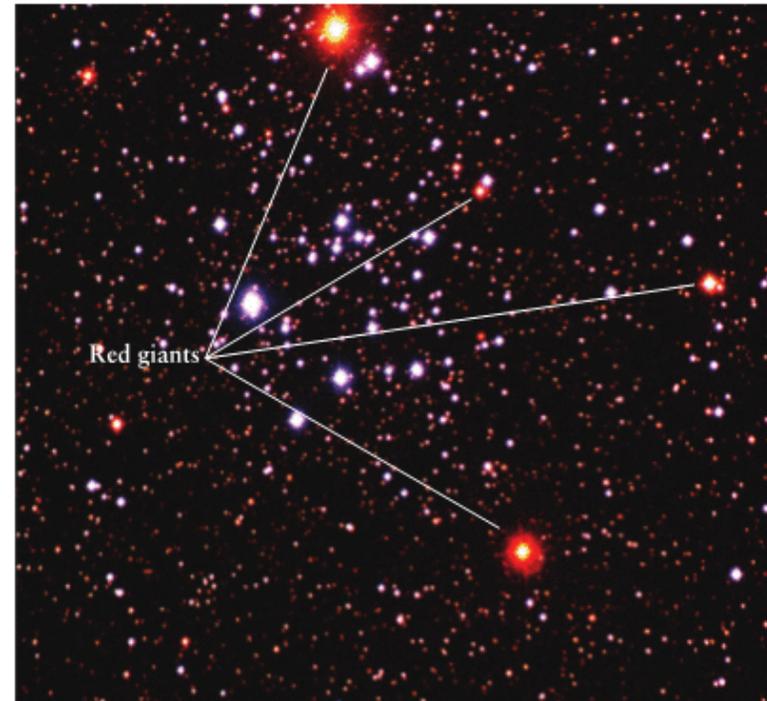
While the core is being compressed the **outer layers expand by the increased luminosity of the Hydrogen burning shell**. As the external layers of gas expand the temperature of this gas decreases (remember example of blowing air through a small opening).

When the temperature of the external layers drops to $\sim 3,500$ K the star appears reddish at which point it has become a **red giant**.

Stellar Evolution: On and After the Main Sequence



(a) The Sun today and as a red giant



(b) Red giant stars in the star cluster M50 R I V U X G

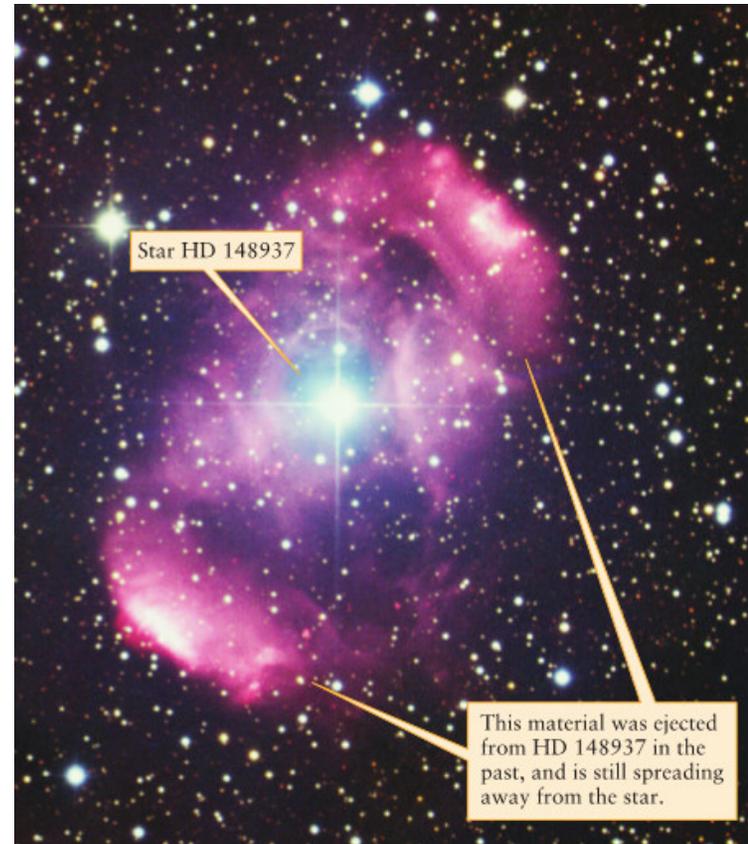
When the sun becomes a red giant its diameter will increase from ~ 0.01 AU to 1 AU.

Stellar Evolution: On and After the Main Sequence

Red giants lose a substantial amount of gas from the outer layers.

The reason for this is that the outer layers have expanded far from the center and since *force* is proportional to $1/R^2$ the force keeping the gas bound is substantially weaker.

Mass loss from red giants is detected from blue-shifted emission lines that imply wind velocities of 10km/s. A red giant may lose $\sim 10^{-7} M_{\odot}$ per year (the sun loses $\sim 10^{-14} M_{\odot}$ per year).



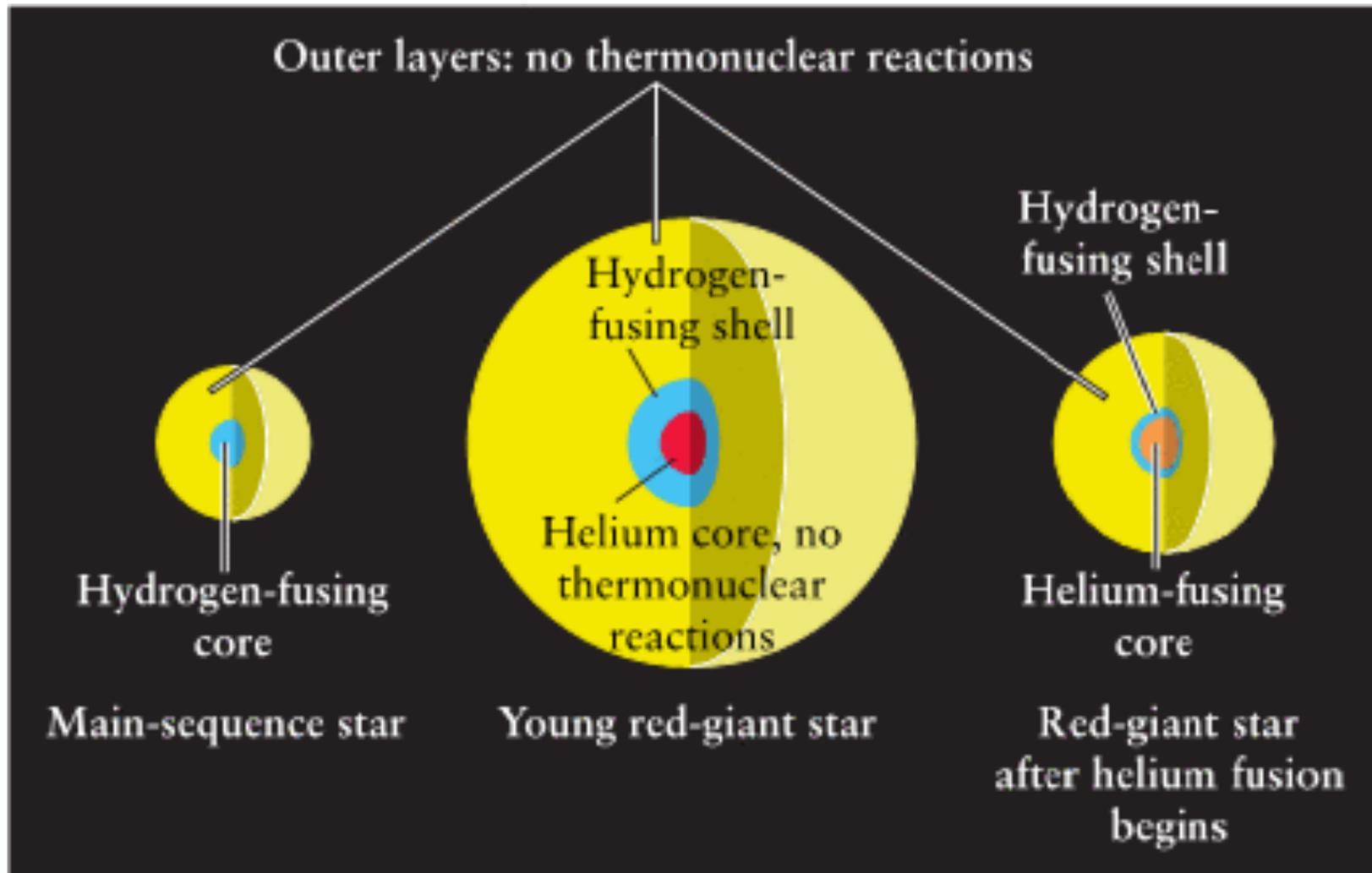
Winds from red giant
HD 148937

The Sun's Fate

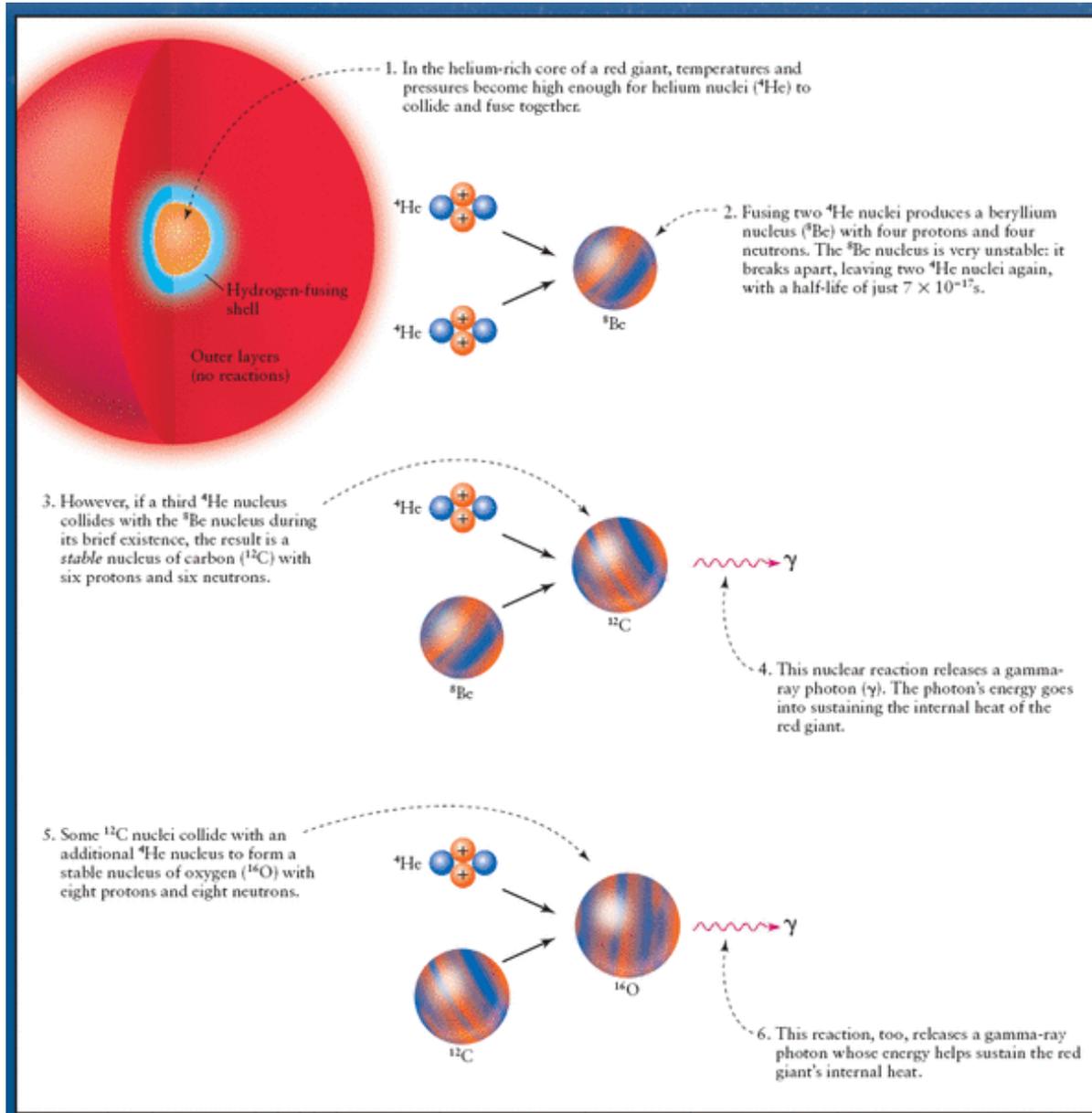
While still on the main sequence the Sun's luminosity will continue to increase. In about 3.5 billion years the average temperature on Earth will be $> 100\text{ }^{\circ}\text{C}$ and the water on the surface will boil away.

H fusion reactions in the core are expected to cease in about 7.5 billion years from now! At about ~ 7.7 billion years from now the Sun will have expanded to a diameter of about 1 AU.

Stellar Evolution: On and After the Main Sequence



Core Helium Fusion: The Triple Alpha Process



Core Helium Fusion

The way He fusion begins in the core of a star depends on its mass.

for $0.4 M_{\text{solar}} < M_{\text{star}} < 2-3 M_{\text{solar}}$ **Helium flash**

$M_{\text{star}} > 2-3 M_{\text{solar}}$ we have gradual ignition of He

Safety Valve of Fusion Reaction Rate in Ideal Gases

In most cases the gases of a star obey the law: $VT^{\gamma-1} = \text{constant}$, where V =volume, T =temperature and γ is the adiabatic index that depends on the type of gas ($\gamma= 1.4$ for diatomic gas). **When the gas expands the temperature goes down.**

When the rate of fusion reactions increases, the outward radiation pressure also increases and expands the core. This expansion results in a decrease in temperature and the reaction rate goes down. This provides a **safety valve** for the star.

Conversely, a decrease in the reaction rate causes a compression of the gas resulting in an increase of the core temperature and the reaction rate goes up.

Degeneracy Pressure

Closely packed electrons resist compression. The reason for this is that no two electrons too close to each other can have the same four quantum numbers n , l , m_l , m_s (**The Pauli exclusion principle**).

The pressure of the electrons resisting compression is called **degeneracy pressure**.

In a red giant with $0.4M_{\odot} < M < 2 - 3 M_{\odot}$ the core must be compressed tremendously in order to become hot enough for helium fusion to begin. **Electron degeneracy pressure** prevents a low mass red giant from compressing any further.



Wolfgang Pauli

Helium Flash

Helium flash is the sudden beginning of helium fusion in the core of intermediate mass stars of less than about 2.25 solar masses.

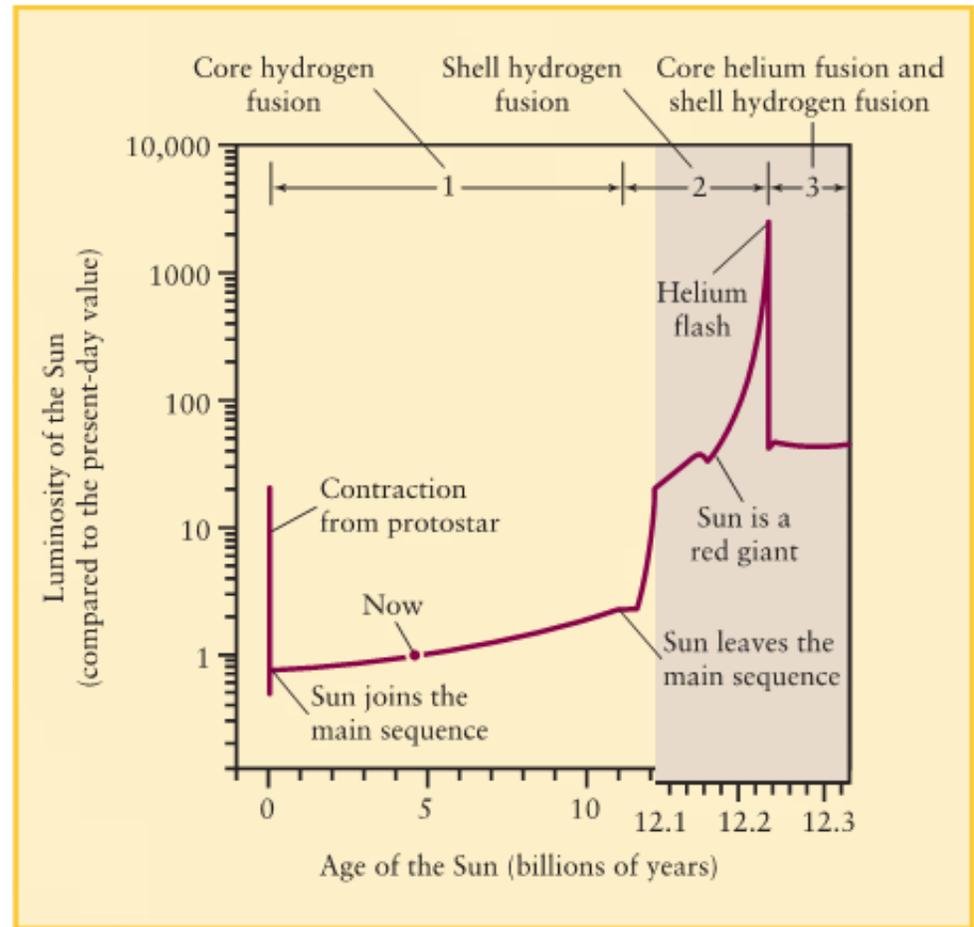
The cores of $0.4 M_{\text{solar}} < M_{\text{star}} < 2-3 M_{\text{solar}}$ red giant stars are degenerate. A helium flash occurs when fusion begins in a degenerate gas.

The cores of $M_{\text{star}} > 2-3 M_{\text{solar}}$ red giant stars are not degenerate. In these stars He fusion begins gradually with no flash.

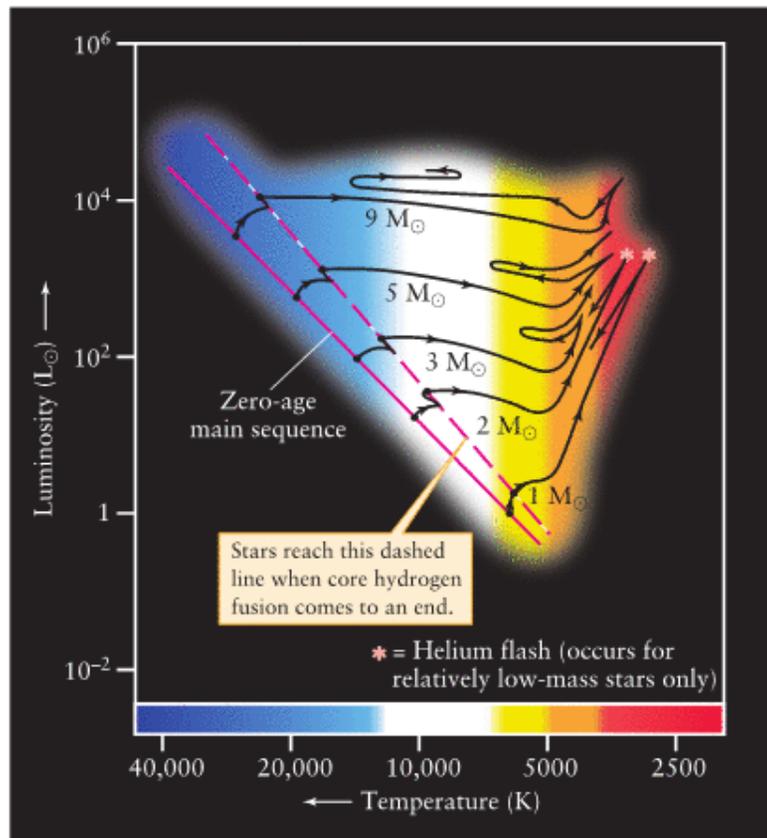
Stellar Evolution: On and After the Main Sequence

Before the He flash the core temperature increases due to compression of the core making the **H fusion reaction rate in the shell increase** leading to a larger luminosity.

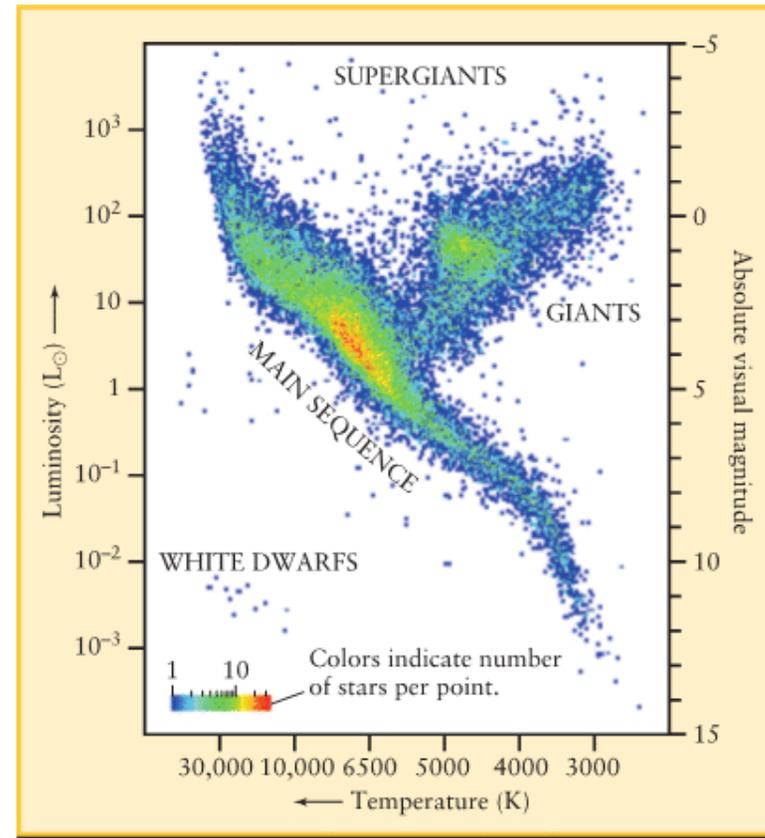
After the He flash the core gas becomes *an ideal gas* and expands leading to a decrease in core temperature. As a result the H shell fusion reaction rate decreases leading to a lower luminosity.



Stellar Evolution: On and After the Main Sequence



(a) Post-main-sequence evolutionary tracks of five stars with different mass



(b) H-R diagram of 20,853 stars—note the width of the main sequence

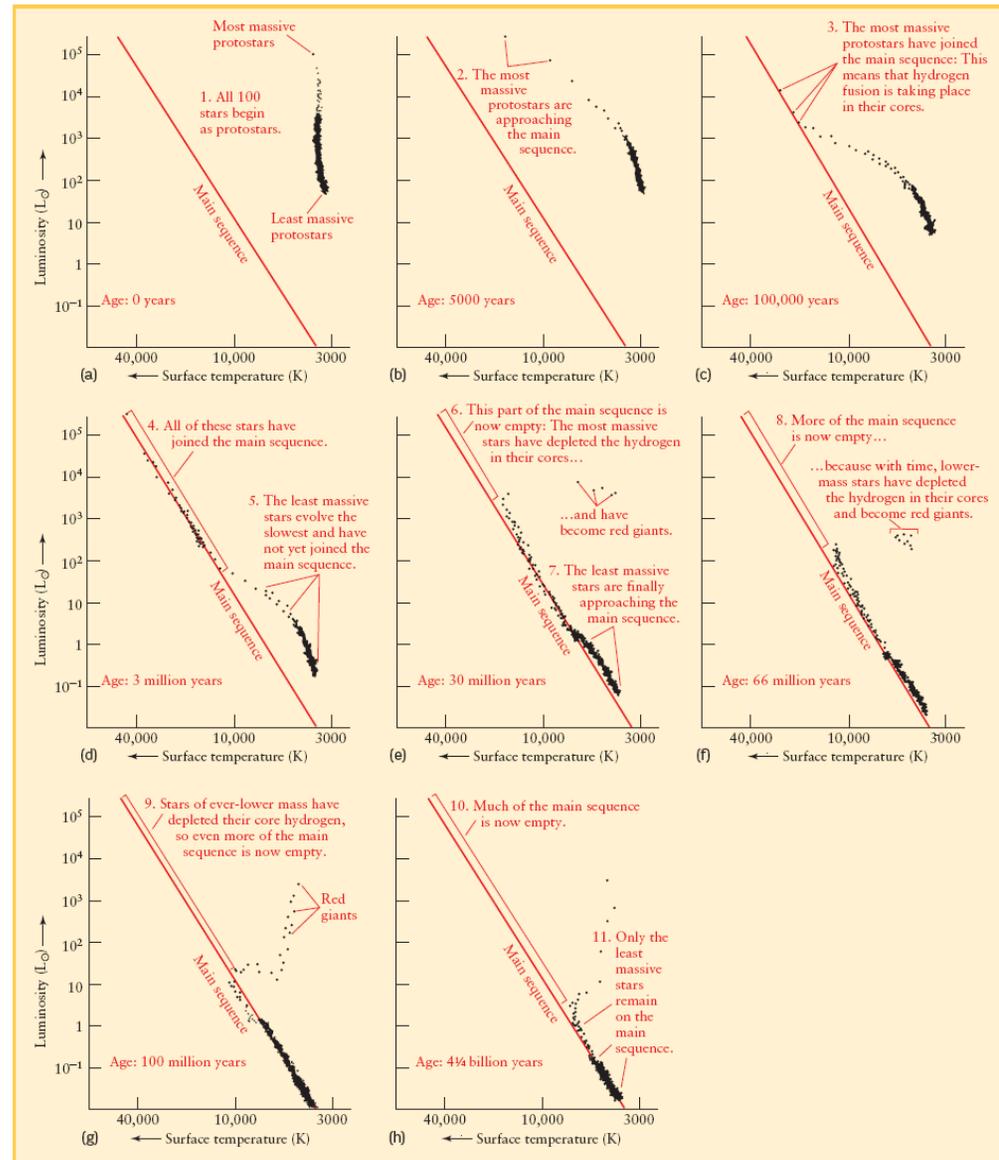
The line of the HR diagram where stars first enter the main sequence is called the **zero-age main sequence**. As H fusion continues in the core the star's luminosity increases and the star's outer surface expands (decrease of stars temperature). Stars positions slowly move away from the ZAMS.

Stellar Evolution: On and After the Main Sequence

The **evolution of a star cluster** can be viewed through the simulations.

This simulation assumes 100 stars of different masses that formed at the same time.

The time it takes for a protostar to reach the Main Sequence and leave it depends on its mass.

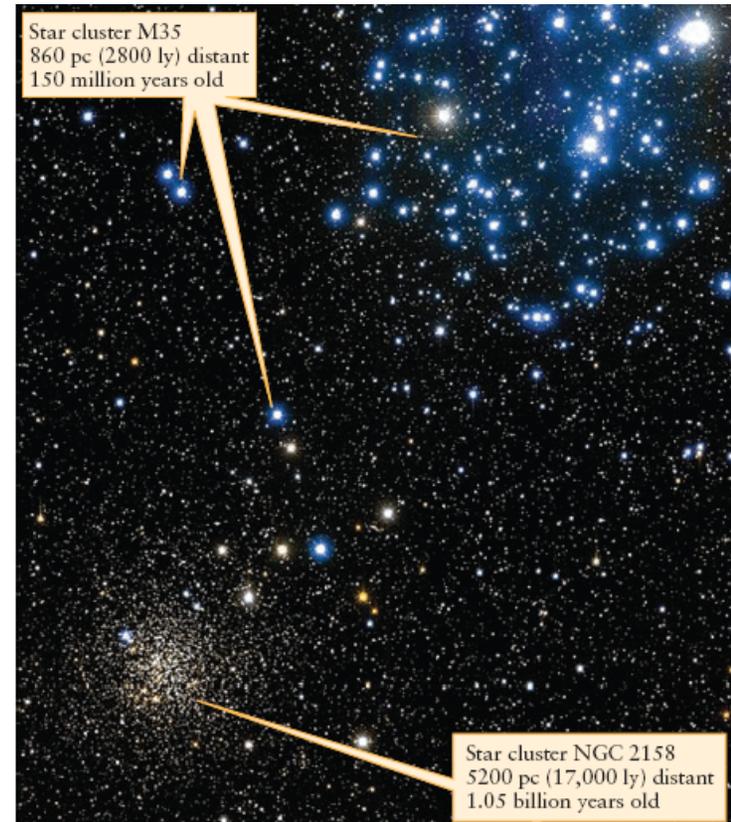


Ages of Open Star Clusters

Open star cluster M35 has many high mass blue stars on the main sequence. It also contains several bright red giant stars. Based on the simulations the cluster is $\sim 10^8$ years old.

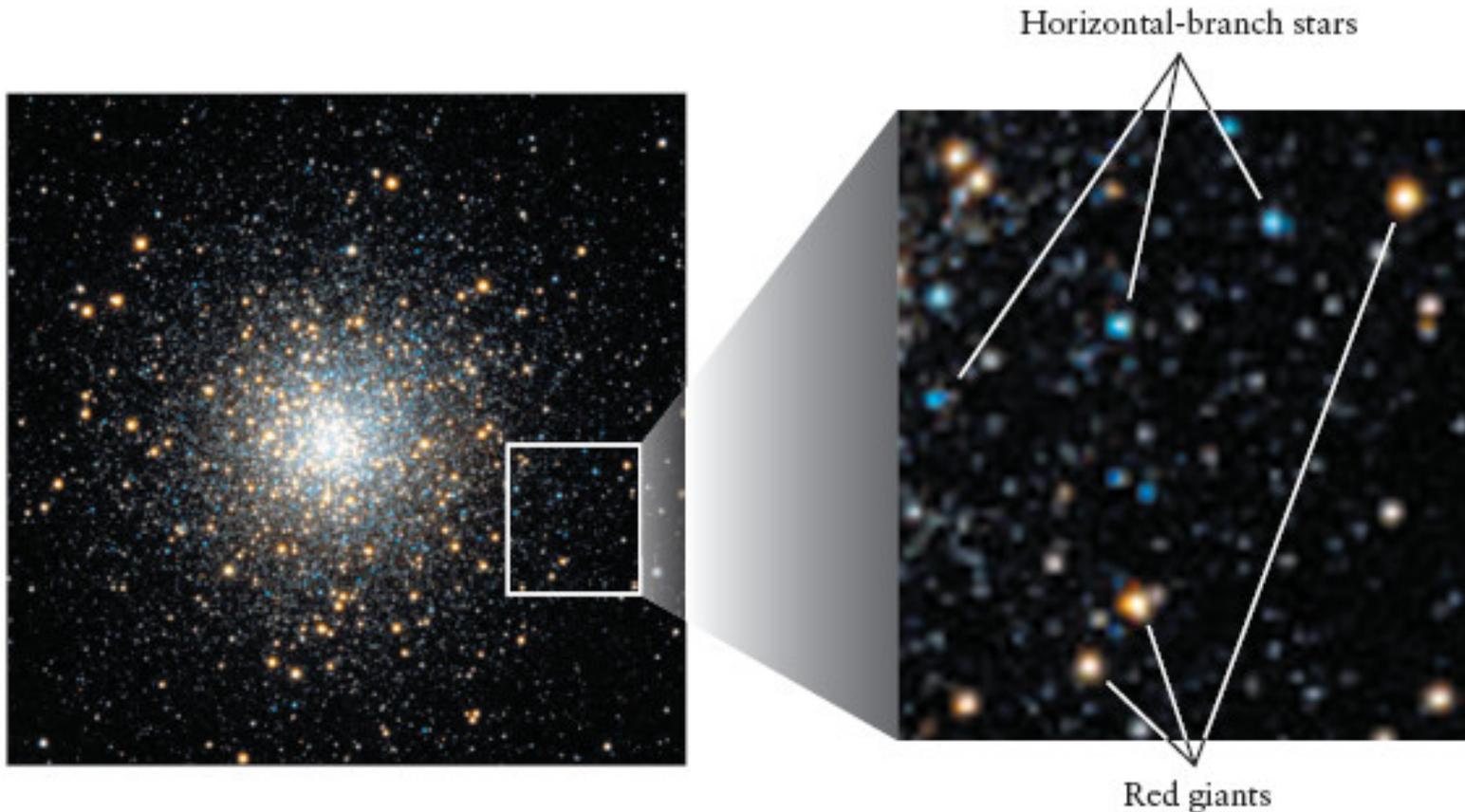
Open star cluster NGC 2158 contains no massive MS blue stars and is $\sim 10^9$ years old.

As a cluster ages, it generally becomes redder in its average color.



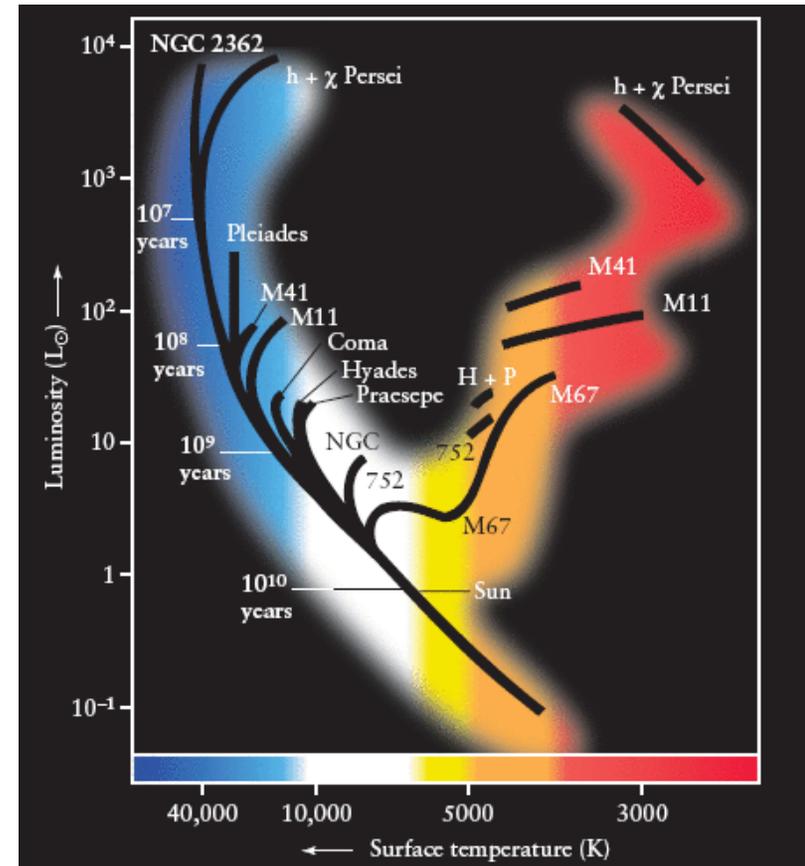
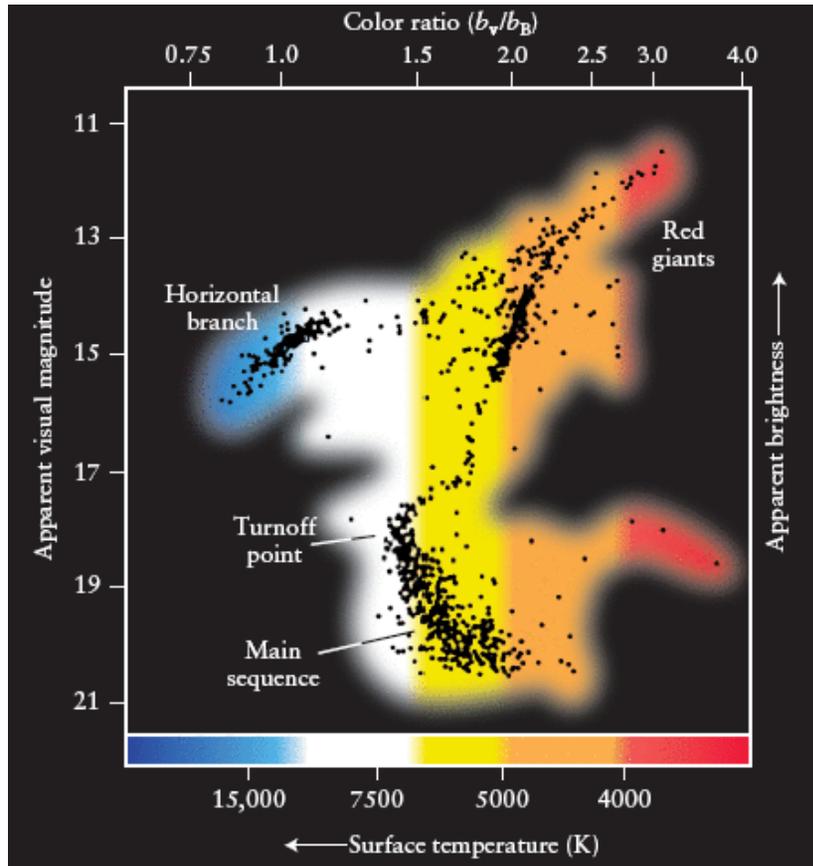
Open star clusters M35 and
NGC 2158

Ages of Globular Star Clusters



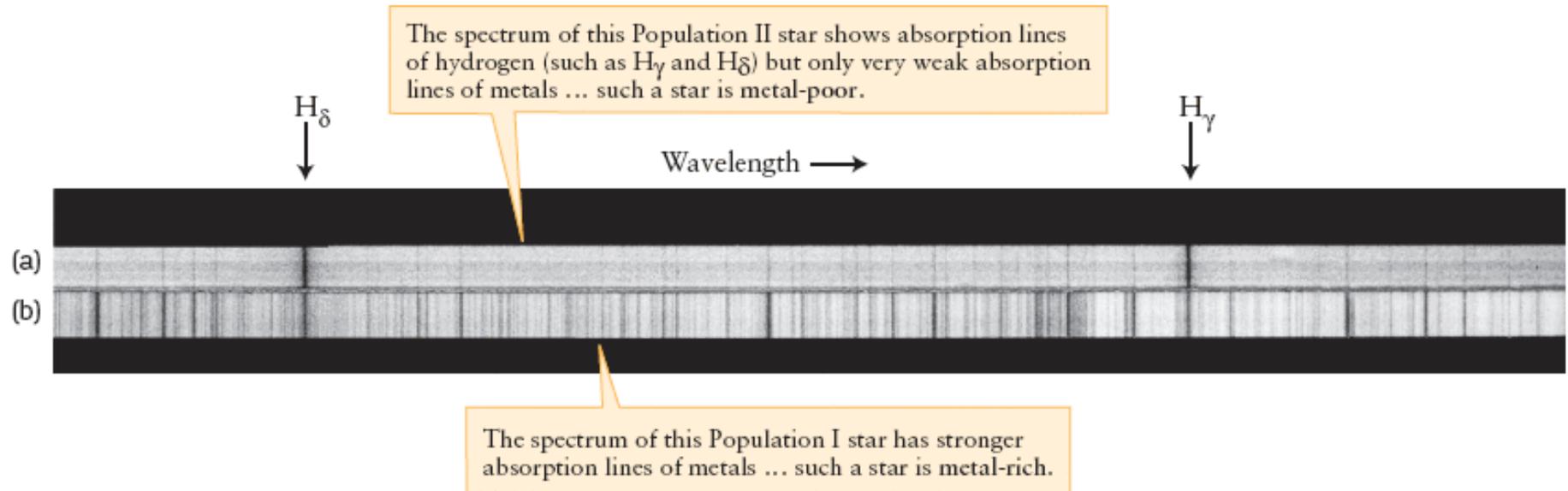
Globular star cluster M10 contains $\sim 10^6$ stars. M10 does not contain any massive main sequence stars and is therefore very old. Low mass stars after going through a He flash, lie on the **horizontal-branch** where they burn He in the core and H in a shell. These stars have relatively hot surface temperatures and look blue.

Ages of Stellar Clusters



(left) A **color - magnitude** diagram for star cluster M55 obtained from star colors (b_V , b_B) and apparent magnitudes. Why is this equivalent to a HR diagram? (right) HR diagrams of several open star clusters listing their **turnoff points** which indicate their ages.

Stellar Populations



Stars in younger star clusters are in general **metal rich** and are referred to as **Population I stars**.

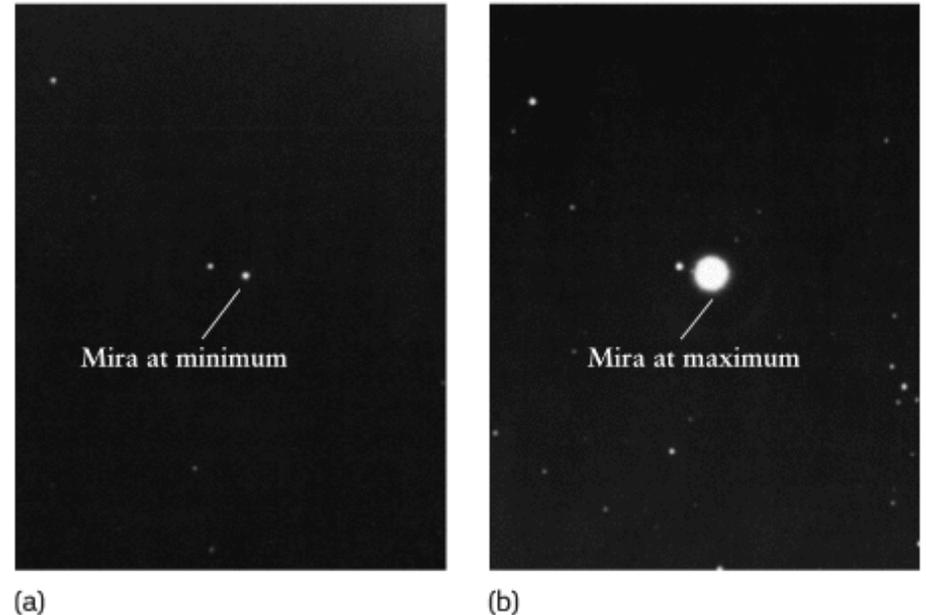
Star in older star clusters (globular clusters) are usually **metal poor** and are referred to as **Population II stars**.

The first stars that formed from primordial gas were metal poor.

Variable Stars: Long-Period Variable

Many stars are found to pulsate in size and brightness. These stars are called **pulsating variable stars**.

Long-period variables are pulsating cool red giants that vary in brightness by a factor of 100 over a period of months to years. Typical surface temperatures of long-period variables are $\sim 3,500\text{K}$ and luminosities of $10\text{-}10,000 L_{\text{solar}}$.

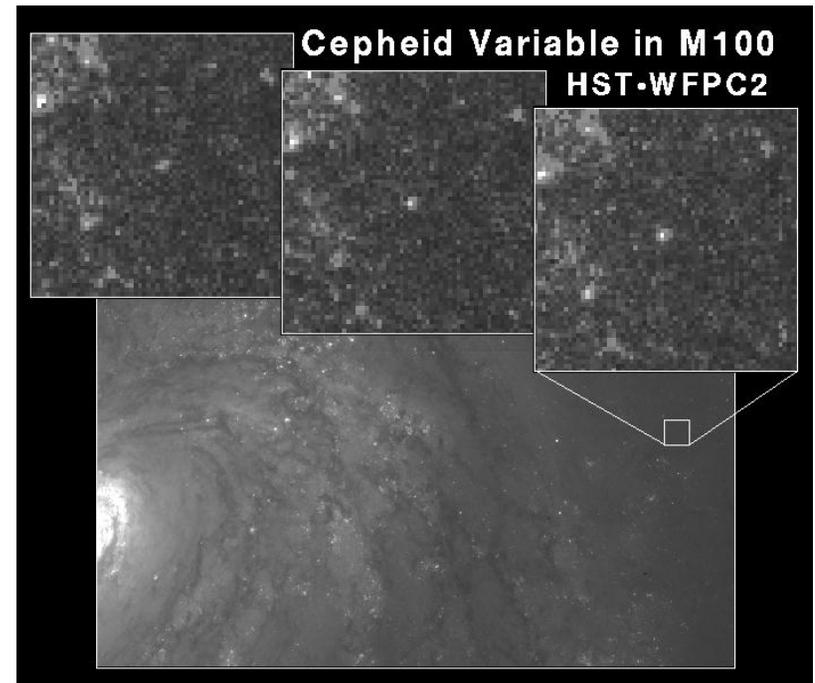


The first **long-period variable** star discovered was Mira (Period ~ 332 days).

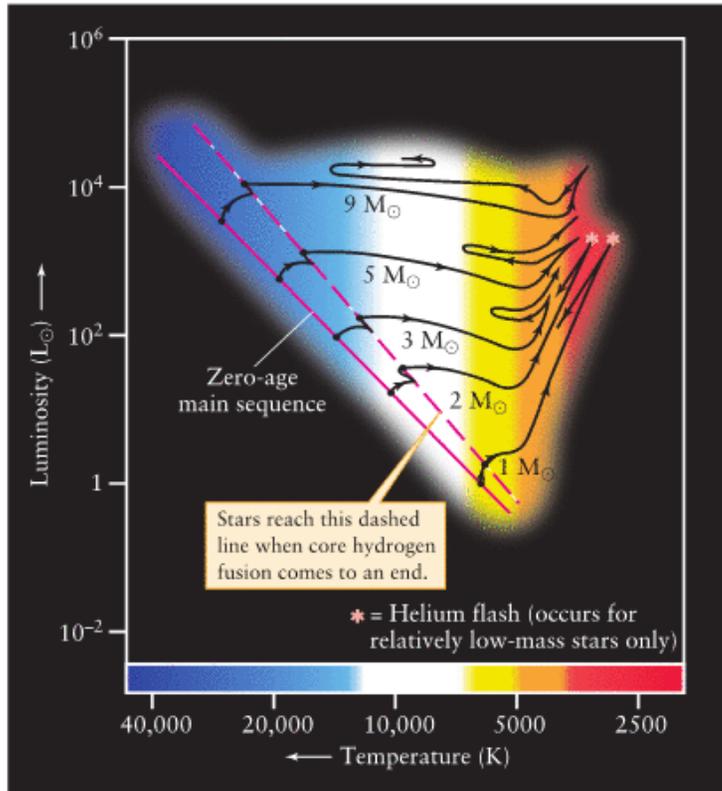
Variable Stars: Cepheid Variables

Pulsating supergiant stars that exhibit rapid brightening followed by gradual dimming with periods ranging from a few to a hundred days are called **Cepheid variables**.

The first Cepheid variable discovered was δ Cephei.

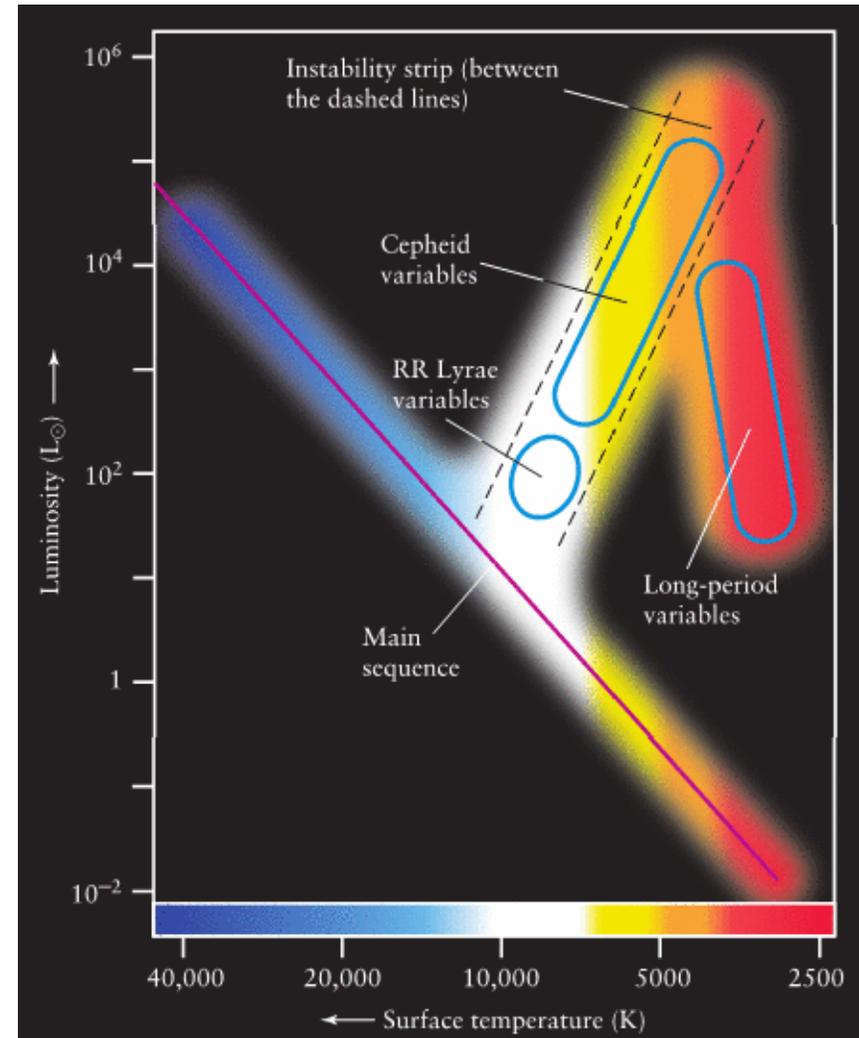


Variable Stars: Cepheid Variables

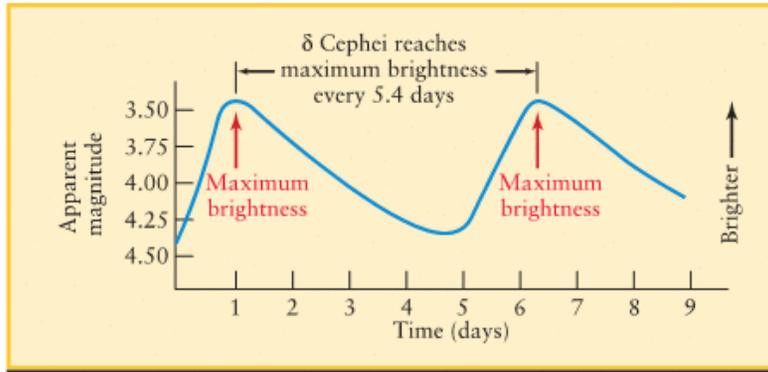


(a) Post-main-sequence evolutionary tracks of five stars with different mass

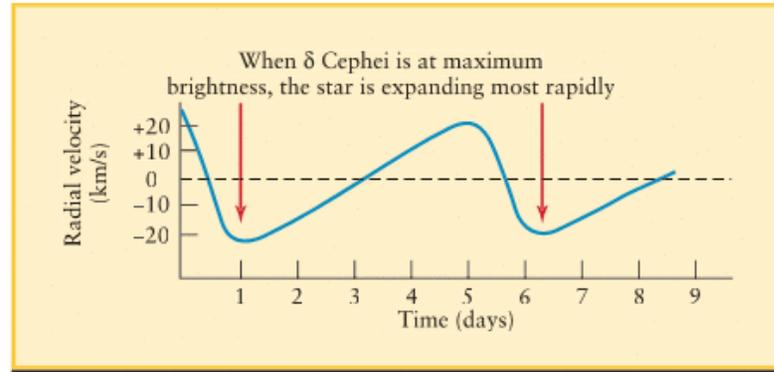
After He fusion begins stars move across an instability strip near the middle of the H-R diagram where they begin to pulsate.



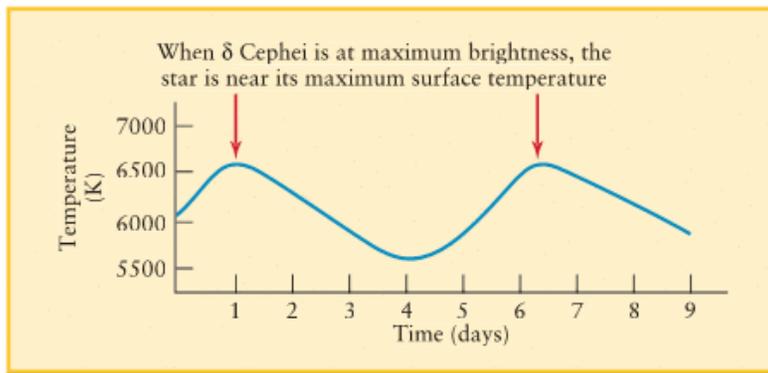
Variable Stars: Cepheid Variables



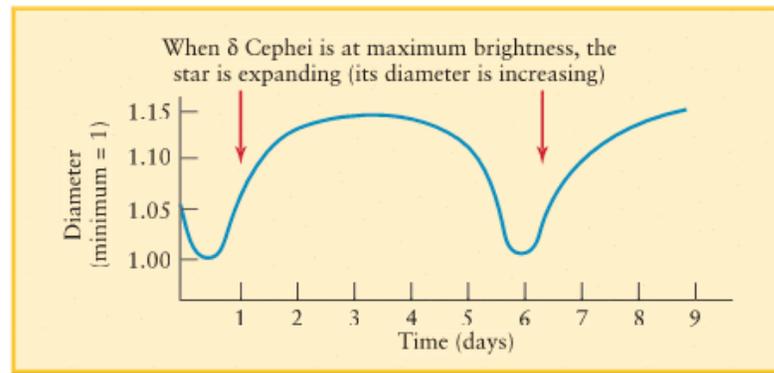
(a) The light curve of δ Cephei (a graph of brightness versus time)



(b) Radial velocity versus time for δ Cephei (positive: star is contracting; negative: star is expanding)



(c) Surface temperature versus time for δ Cephei



(d) Diameter versus time for δ Cephei

Variations in brightness and temperature of δ Cephei a star that lies in the instability strip. The variations occur because the star's outer envelope cyclically expands and contracts.

Variable Stars: Cepheid Variables

Why Cepheids vary:

When a Cepheid variable pulsates, the star's surface oscillates up and down. During these cyclical expansions and contractions, the star's gases alternately heat up and cool down.

Arthur Eddington suggested that a Cepheid pulsates because the **star is more opaque when compressed** than when expanded.

When the star is compressed and opaque, heat cannot easily escape increasing the internal pressure that pushes the star surface outward.

John Cox showed that under certain conditions He in the outer layers of a star becomes ionized and opaque when compressed.

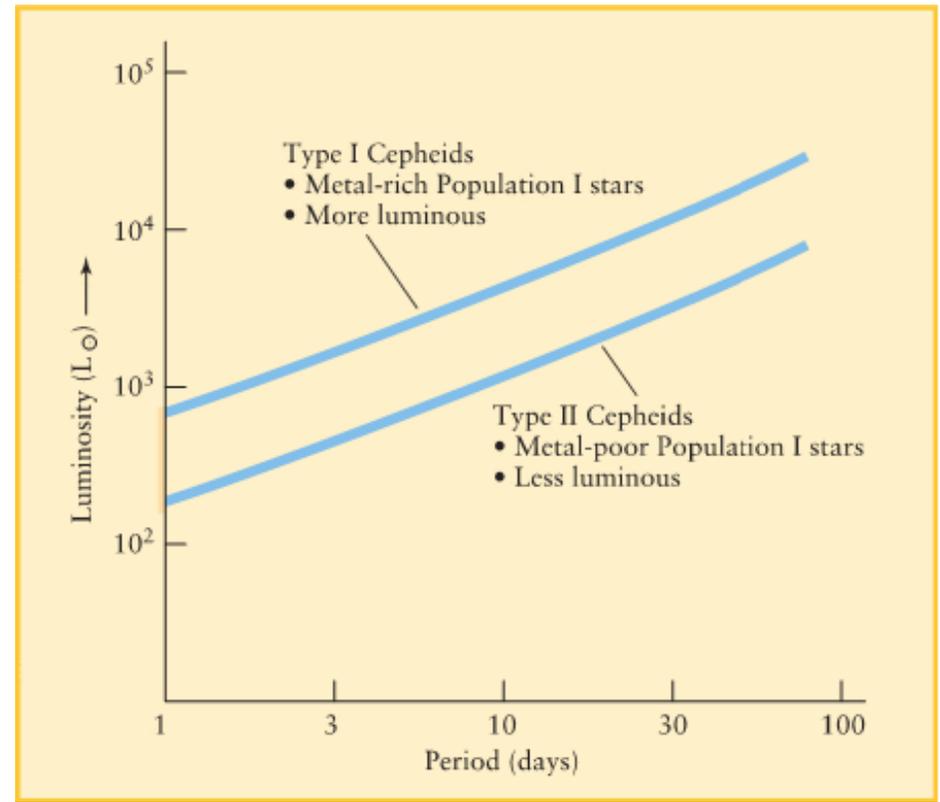
Cepheid Variables: Period – Luminosity Relation

These **L-P relations** together with the brightness of a Cepheid are **used to infer its distance**.

The abundance of metals in a Cepheid's outer layers plays a significant role on how it pulsates.

Metal rich Cepheids are called **Type I Cepheids**.

Metal poor Cepheids are called **Type II Cepheids**.



The period of a Cepheid's pulsations is correlated to its average luminosity.

Variable Stars: RR Lyrae Variables

RR Lyrae variables are pulsating horizontal branch stars of spectral class A (and rarely F), with a mass of around half the Sun's. Their periods are less than one day and they are commonly found in globular clusters.

RR Lyrae stars are old, relatively low mass, metal-poor "Population II" stars.

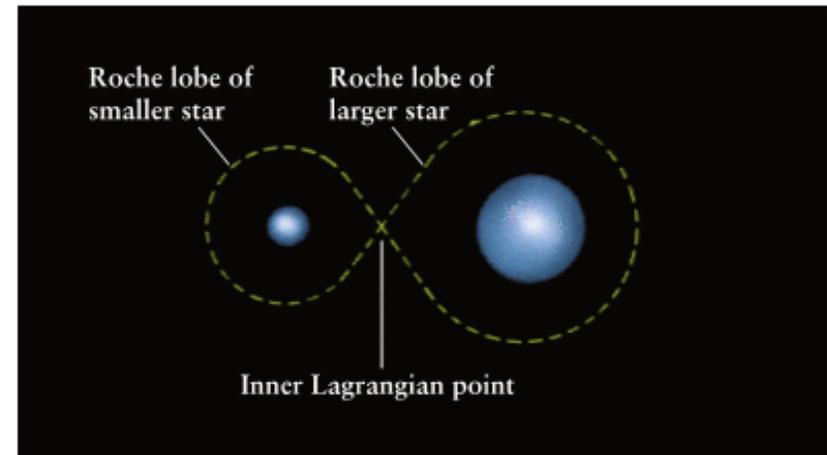
RR Lyrae stars are named for their prototype RR Lyrae in the constellation Lyra.

Stellar Evolution in Close Binaries

A red giant in a close binary system may **transfer mass** to its companion star and alter the evolution of both stars.

Roche lobe: A teardrop-shaped volume surrounding a star in a binary inside which gases are gravitationally bound to that star.

If gas from the outer layers of a red giant moves beyond its Roche lobe it is not bound and may be transferred to the companion star.

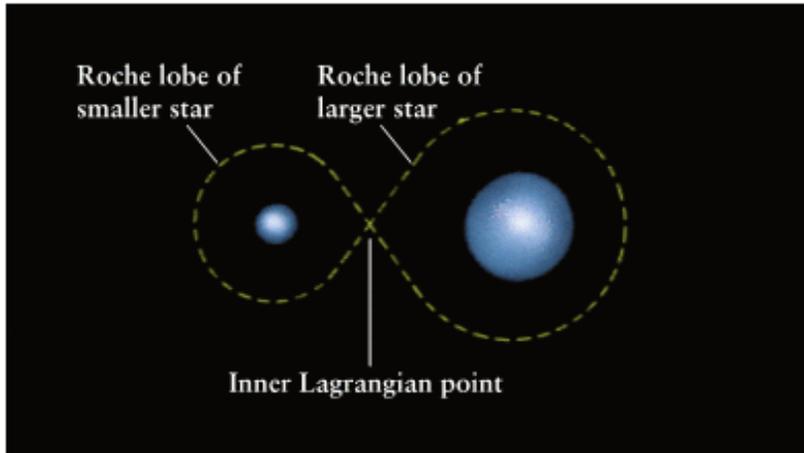


(a) Detached binary: Neither star fills its Roche lobe.

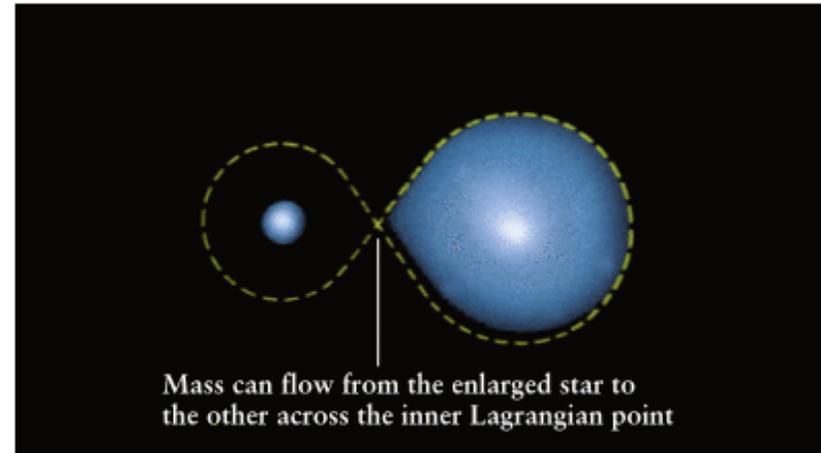
The point where the two Roche lobes touch is called the **inner Lagrangian point**.

If a star does not fill its Roche lobe it is called **detached**.

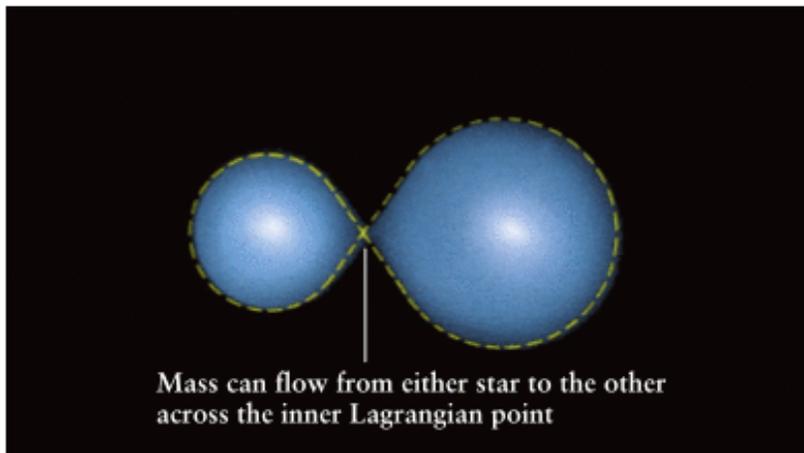
Stellar Evolution in Close Binaries



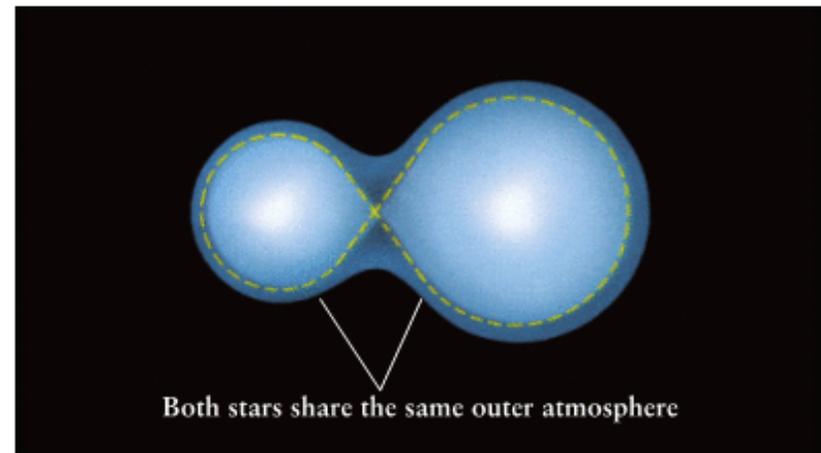
(a) Detached binary: Neither star fills its Roche lobe.



(b) Semi-detached binary: One star fills its Roche lobe.



(c) Contact binary: Both stars fill their Roche lobes.



(d) Overcontact binary: Both stars overfill their Roche lobes.

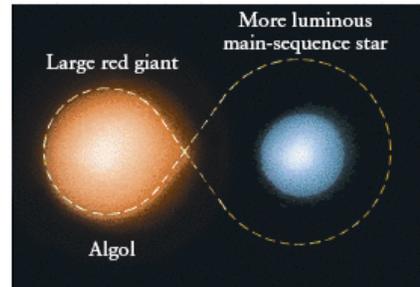
Close Binary Star Systems

Stellar Evolution in Close Binaries

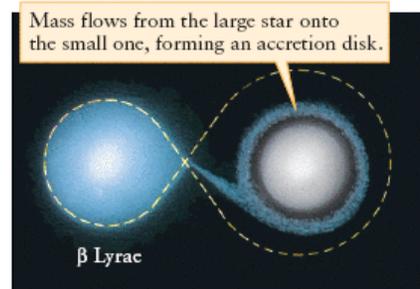
Algol : an eclipsing close binary with the detached star being more luminous more massive and on the main sequence.

β Lyrae: an eclipsing close binary with the detached star being more massive and surrounded by an accretion disk formed by material transferred from the companion star. The disk obscures the star.

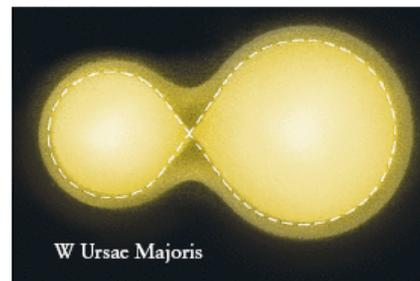
When the detached star in a β Lyrae or Algol system fills its Roche lobe it becomes an overcontact binary like **W Ursae Majoris**.



(a) A semidetached binary



(b) A semidetached binary with mass transfer



(c) An overcontact binary

